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WELDED CONTINUOUS FRAMES AND THEIR COMPONENTS

NOMOGRAPHS FOR THE SOLUTION OF BEAM-COLUMN PROBLEMS

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List

by

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SYNOPSIS

The principal purpose of this paper is to present nomographs and charts which are useful in the solution of a wide variety of beam-column problems.

Methods are presented for the determination of the load-deflection behavior of restrained beam-columns of constant cross section. The restraint may be elastic or elastic-plastic; failure is assumed to take place by excessive bending in the plane of the applied moments. Several example problems illustrate the application of the charts.

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I. INTRODUCTION

Columns in framed structures are connected at their ends to beams and other columns. These elements provide translational and rotational restraints to the columns.

The columns deform from the very beginning of loading due to the bending moments transmitted from the beams through the rigid beam-to-column connections. When the axial load in a column becomes large enough, the sign of the bending moments at the column ends may be reversed, and the beams take over the role of restraining the column and thereby they limit column deflection. It is thus seen that the behavior of restrained columns must be studied in order to arrive at a more accurate estimate of the strength of columns in continuous structures than would be obtained by assuming pinned ends.

The ensuing report will be concerned with the determination of the behavior of the restrained columns which fail by excessive bending about one of the principal axes of the cross section. The bending moment, the restraint, and all the deformations occur in one and the same plane. It is assumed that the column ends do not translate. The method presented here is based on theoretical work developed in Refs. 1 and 2. This research systematized the approach used by Chwalla[3] and extended its applicability to cases where the rotational restraints at the ends of the column are not elastic. The results of the research are presented in the form of nomographic charts which are given in the Appendix.

The nomographs in the Appendix permit the solution of the following problems for as-rolled wide-flange steel beam-columns bent in the plane of the web:

1) Beam-columns with symmetric end moments and end restraints.
2) Beam-columns with one end pinned.
3) Pinned-end columns under any combination of end moments.

Example problems are worked out with the aid of the charts. Moment-versus-end rotation curves for wide-flange sections are also given in the Appendix. These M-θ curves are presented for strong axis as well as weak axis bending.

II. DEVELOPMENT OF THE NOMOGRAPHS

II.1 Construction of the Column Deflection Curves

The most general case of the beam-column problem without translating ends is shown in Fig. 1(a). The compressive load P is applied to each end with different eccentricities e_A and e_B. In addition to the applied moments $P \cdot e_A$ and $P \cdot e_B$ there are restraining moments at the ends. These moments depend on the rotation characteristics of the supports. These moments are denoted by the symbols $f_A(\theta_A')$ and $f_B(\theta_B')$ to indicate that they are functions of the end rotations.

A member such as column AB in Fig. 1(a) may be considered as a segment of a Column Deflection Curve[1,2,3]. A Column Deflection Curve is defined as the shape that a compressed member would take if held in a bent configuration by axial loads applied to the ends. For the column deflection curve shown in Fig. 1(b) the following equations are obtained from the geometry of the deflection curve:

$$\theta_A' = \theta_A - \alpha \tag{1}$$

$$\theta_B' = \theta_B + \alpha \tag{2}$$

$$\alpha = \frac{y_A - y_B}{L} \tag{3}$$

$$e_A = \frac{f_A(\theta_A') - Py_A}{P} \qquad (4)$$

$$e_B = \frac{f(\theta_B') - Py_B}{P} \qquad (5)$$

$$e_A/e_B = \frac{f_A(\theta_A') - Py_A}{f_B(\theta_B') - Py_B} \qquad (6)$$

Values for θ_A', θ_B', α, y_A, y_B, e_A, and e_B are positive when they are shown as in Fig. 1(a) and (b). The bending is positive when the upper fibers of the column in Fig. 1(b) are in tension.

It is apparent from the description of the column deflection curve that an infinite number of such curves are possible for a given column cross section, stress-strain diagram, and average compressive stress. Each of these may be conveniently identified by θ_o or y_m (where θ_o is a given initial slope and y_m is a deflection at the midspan of the column deflection curve). The column deflection curves have been constructed for given axial load ratios P/P_y and initial slopes θ_o by a numerical integration of the moment-curvature curves. The details of the numerical integration process, as well as the various theoretical implications associated with the column deflection curves are given in Refs. 1 and 2.

The moment curvature relationships used in the construction of the column deflection curves were taken from the M-ϕ curves developed in Ref. 4. The bases for these curves are an elastic-fully plastic stress-strain diagram (with the yield stress σ_y = 33 ksi and the modulus of elasticity E = 30,000 ksi), a linearly varying symmetric residual stress pattern (with the maximum compressive residual stress of 0.3 σ_y occurring at the flange tips), and a typical wide-flange shape (8WF31).

II.2 Development of the Nomographs

(a) Columns with Equal End Moments and Equal End Restraints

Figure 2(a) shows a symmetrically loaded and symmetrically restrained column, while Fig. 2(b) shows the column deflection curve that includes the column of Fig. 2(a). The segment of the column deflection curve A'-B' corresponds to the restrained column A-B. At point A or B in Fig. 2(a) the equality of internal and external moments requires that

$$P \cdot y = f(\theta_A') - P \cdot e = M_A' \tag{7}$$

Because of the symmetry, the end rotation of the column will be equal to the slope θ' at A' or B' on the column deflection curve.

Figure 3 shows how the information from one column deflection curve is plotted for the nomograph. This nomograph correlates the slope θ', the moment $M = P \cdot y$, and the length L of the column segment at any point B' on the column deflection curve shown in the upper right hand portion of Fig. 3. The upper nomographic curve shows the M-θ' relationship, and the lower curve represents the $L - \theta'$ curve. The moments, slopes and lengths for two typical points B_1' and B_2' are shown by the dashed lines. Because of symmetry only one half the column length is plotted in the lower curve. The complete nomograph for a particular value of P/P_y is constructed from several column deflection curves (identified by different values of θ_o) in the same manner, and these nomographs are shown in Appendix VII.1.

The nomographs for the several values of P/P_y are based on the properties of the 8WF31 section, and thus they are strictly applicable to this section only. They have been non-dimensionalized however, so that use can be made of them for all rolled wide-flange sections normally used as columns. When the nomograph is used for sections other than the 8WF31, the error will be small because the distribution of the areas for all wide-flange column sections about the neutral axis is similar. In fact, the results based on an 8WF31 section nearly always will give conservative values for the strength of a rolled-wide-flange column because this section has one of the more unfavorable thrust-moment-curvature relationships of the wide-flange column sections rolled.[5]

(b) Column with One End Pinned

The pinned end of the column must always correspond to the origin of the column deflection curve. In Fig. 4a such a column is shown. One end of the column is pinned (A) and the other end, to which also the external moment is applied, is restrained (B). The corresponding segment of the column deflection curve is given in Fig. 4b. The way in which the nomograph is constructed may best be understood by a consideration of Fig. 5. This figure shows how the information from a single column deflection curve is organized. In the upper section the internal moment at a point B' is plotted against the value of θ'. Coordinates of typical points B' are shown in the upper (M-θ' curves) and lower portion (L-θ' curves) of the nomographs.

More details on the nomographs, including their use for finding various relationships (M-θ, L/r-θ, etc.) are discussed in Refs. 1 and 2. The nomographs in Appendix VII.1 are for the symmetrical and the pinned-end loading cases discussed above. They are applicable for strong-axis bending only. Nomographs are given for both loading cases for $P = 0.12\ P_y$, $0.2\ P_y$, $0.3\ P_y$, $0.4\ P_y$ and $0.6\ P_y$. The curves are plotted on a rectangular grid-system, thus permitting the solutions of problems graphically by the use of transparent overlays.

III. USE OF THE NOMOGRAPHS

In the following section of this report the use of the information contained in the nomographic charts is illustrated by several example problems.

III.1 Restrained Columns with Equal End Moments and Equal End Restraints

(a) Determination of the maximum length L

Find the maximum length L of span A-B (Fig. 6). Consider the cases when s/r = 56.5 and 113.

Given: (1) The rolled steel wide-flange column A-B of Fig. 6 with symmetrical restraining spans A'-A and B'-B.

(2) $P/P_y = 0.3$

(3) $\sigma_{rc} = 0.3\,\sigma_y$

(4) $M_p = 1.11\,M_y$

(5) $e = e_A = e_B = 2.88\,r$

(6) $d/r = 2.3$ (approximately constant for all standard rolled steel column section)

(7) $E = 30 \times 10^3$ ksi, $\sigma_y = 33$ ksi

<u>Solution</u>: The restraining functions at A and B are approximated according to simple plastic theory, that is, the restraining beams are assumed to remain entirely elastic up to the formation of a plastic hinge.

$$\theta_A = \frac{M_A s}{3EI} \quad \text{for } \theta_A \leqslant \theta_p$$

For $\quad \theta_A = \theta_p, \; M_A = M_p$

$$\theta_p = \frac{1.11\ M_y s}{3EI} \qquad M_y = \sigma_y \cdot I \cdot 2/d$$

Then $\quad \theta_p = 0.00036 \text{ s/r}$

and thus

$$f(\theta_A) = \frac{M_p}{\theta_p} \cdot \theta = 3080(r/s)M_y\ \theta, \text{ for } \theta \leqslant 0.00036 \text{ s/r}$$

$$f(\theta_A) = M_p, \text{ for } \theta \geqslant 0.00036 \text{ s/r}.$$

The external moments at A and B, $M_e = f(\theta) - Pe$, acting on column A-B must equal the internal moments at these points. In nondimensional terms:

$$M/M_y = 3080\,(r/s)\,\theta - 1.0 \quad \text{when} \quad \theta \leqslant 0.00036 \text{ s/r} \quad \text{(a)}$$

$$M/M_y = 1.11 - 1.0 = 0.11 \text{ when} \quad \theta \geqslant 0.00036 \text{ s/r} \quad \text{(b)}$$

The expression given by Eqs. (a) and (b) is plotted in the upper portion of the appropriate nomograph of Appendix VII.1 (as shown for the case of $P/P_y = 0.3$ in Fig. 7), and the intersections with the M-θ curves of the θ_o^1, θ_o^2,...θ_o^n column deflection curves are carried down to the lower portion of the nomograph to give lengths $L_o^1/2r$, $L_o^2/2r$, ... $L_o^n/2r$ representing equilibrium configurations of the column. By connecting these points in the lower portion one obtains the relationship between the slenderness ratio L/r and the end rotation at A and B for each equilibrium configuration.

From the lower portion of the nomograph the maximum value of L/r consistent with equilibrium for each value of s/r is indicated by an arrow in the table belwo.

s/r	θ_o	L/2r	L/r	
56.5	0.01	--	--	
	0.02	95	190	
	0.025	100	200	maximum
	0.030	96	192	
	0.035	94	188	
	0.040	90	180	
	0.050	73	146	
113	0.035	82.5	165	
	0.040	87.5	175	maximum
	0.050	74.0	148	

Answers: $(L/r)_{max} = 200$ for $s/r = 56.5$

$(L/r)_{max} = 175$ for $s/r = 113$

(b) Determination of the maximum eccentricity e of the problem in Fig. 8.

Given: (1) The rolled steel wide-flange column A-B of Fig. 8 with restraining spans A'-A and B'-B.

(2) $P/P_y = 0.4$

(3) $\sigma_{rc} = 0.3\,\sigma_y$

(4) $M_p = 1.11\ M_y$

(5) $d/r = 2.3$

(6) $E = 30 \times 10^3$ ksi, $\sigma_y = 33$ ksi

Solution: The restraining functions at A and B (in accordance with the approximation of III.1(a)) are

$$f(\theta_A) = f(\theta_B) = f(\theta) = \begin{cases} 54.5\ M_y\theta_A, & \theta \leqslant 0.0204 \\ M_p, & \theta \geqslant 0.0204 \end{cases}$$

A horizontal line in the lower portion of the nomograph is drawn at the specified slenderness ratio ($L/2r = 40$).

Intersections with the curves of the nomographs are carried up to the corresponding nomograph curves in the upper portion. By connecting these points in the upper portion one obtains the relationship between moment and rotations at A and B. This would be the M-θ curve for the corresponding unrestrained beam column.

From the nomograph

$$(M/M_y)_{max} = -0.381 \quad \text{for } L/r = 80$$

$$\theta = 0.0243$$

Note that $\theta \geqslant 0.0204$, $\therefore$ $f(\theta) = M_p$

From Eq. (4)

$$\frac{P \cdot e}{M_y} = \frac{f(\theta)}{M_y} - \frac{M}{M_y}$$

$$= 1.110 - (-0.381)$$

$$= 1.491 \text{ since } \theta = 0.0243 \geqslant 0.0204$$

$$e = \frac{1.491\ M_y}{0.4\ P_y} = \frac{1.491}{0.4} \cdot \frac{r}{d} \cdot 2r$$

$$= 3.24r$$

The maximum eccentricity is thus

$$e_{max} = 3.24r \quad \text{for } L/r = 80, \text{ and } s/r = 57.6$$

III.2 Restrained Columns with One End Pinned

(a) Determination of the maximum length L

Find the maximum length L of span A-B (Fig. 9). Consider the cases when $s/r = 56.5$ and 113.

Given: (1) The rolled steel wide-flange column A-B of Fig.9 with restraint span B-B'.

(2) $P/P_y = 0.3$

(3) $\sigma_{rc} = 0.3\ \sigma_y$

(4) $M_p = 1.11\ M_y$

(5) $e_B = 2.88\ r$

(6) $d/r = 2.3$

(7) $E = 30 \times 10^3$ ksi, $\sigma_y = 33$ ksi

Solution: The restraining functions at B (in accordance with the approximation of III.1(a)) is given by:

$$f(\theta_B') = \begin{cases} 3080\ r/s\ \ M_y\theta_B', & \theta_B' < 0.00036\ s/r \\ M_p, & \theta_B' \geq 0.00036\ s/r \end{cases}$$

The external moment, $f(\theta_B') - Pe_B$ must be equal to the internal moment of the column at B. The equality when non-dimensionalized is:

$$\frac{M}{M_y} = 3080\,(r/s)\,\theta_B' - 1.0 \text{ when } \theta_B' \leq 0.00036\ s/r \quad \text{(a)}$$

$$\frac{M}{M_y} = 1.11 - 1.0 = 0.11 \text{ when } \theta_B' \geq 0.00036\ s/r \quad \text{(b)}$$

The non-dimensionalized function M/M_y is plotted in the upper portion of the appropriate nomograph of Appendix VII.1 (See Fig. 9). The intersections with M/M_y-θ' curve of the θ_o^1, θ_o^2, θ_o^3, ... θ_o^n column deflection curves are carried down to give lengths L_1/r, L_2/r, L_3/r, ... L_n/r representing equilibrium configurations. By connecting these points in the lower portion one obtains the relationship between the slenderness ratios L/r and the end rotations at B for each equilibrium configuration. The construction of the L/r vs θ curve is shown in solid lines in Fig. 9 for s/r = 56.5.

From the lower portion of the nomograph the maximum value of L/r consistent with equilibrium is indicated by an arrow in the following table:

s/r	θ_o	L/r
56.5	0.020	173
	0.030	185 ←
	0.035	181
113	0.030	161
	0.035	166
	0.040	168 ←
	0.050	142

$(L/r)_{max}$ = 185 for s/r = 56.5

$(L/r)_{max}$ = 168 for s/r = 113

(b) Determination of the maximum eccentricity e

Find the maximum eccentricity e (Fig. 10).

Given: (1) The rolled steel wide-flange column A-B of Fig. 10 with restraining span B-B'.

(2) P/P_y = 0.4

(3) $\sigma_{rc} = 0.3\sigma_y$

(4) $M_p = 1.11\ M_y$

(5) $d/r = 2.3$

(6) $E = 30 \times 10^3$ ksi, $\sigma_y = 33$ ksi

<u>Solution</u>: The restraining function at B (in accordance with the approximation of III.1(a)):

$$f(\theta_B') = \begin{cases} 3080\ r/s\ M_y\ \ \theta_B' & , \ \theta_B' \leqslant 0.00036\ s/r \\ M_p & , \ \theta_B' \geqslant 0.00036\ s/r \end{cases}$$

for $s/r = 80$

$$f(\theta_B') = \begin{cases} 38.4\ M_y\ \ \theta_B' & , \ \theta_B' \leqslant 0.0288 \\ M_p & , \ \theta_B' \geqslant 0.0288 \end{cases}$$

A horizontal line in the lower portion of the nomograph is drawn at the specified slenderness ratio (L/r = 100). Intersections with the curves of the nomograph $P/P_y = 0.4$ are carried up to the corresponding nomograph curves in the upper portion. By connecting these points in the upper portion one obtains the relationship between the end moment and end rotation at B for each equilibrium configuration. From Fig. 10 $(M/M_y)_{max} = -0.485$ for $L/r = 100$.

$$\theta_B' = 0.0320 \geqslant 0.0288$$

From the equation of equilibrium (Eq. (4))

$$\frac{P \cdot e}{M_y} = \frac{f(\theta)}{M_y} - \frac{M}{M_y}$$

$$= 1.110 - (-0.485)$$

$$= 1.595$$

$$e = \frac{1.595\ M_y}{0.4\ P_y} = \frac{1.595}{0.4 \cdot 2.3} \cdot 2r$$

$$e/r = 3.47$$

The maximum eccentricity $e_{max} = 3.47r$ for $L/r = 100$, and $s/r = 80$.

III.3 Unrestrained Columns

(a) Unrestrained columns with unequal end moments

Find the maximum length consistent with equilibrium of column A-B in Fig. 11.

Given: (1) The pin ended rolled steel, wide-flange column A-B of Fig. 11 with unequal end moments.

(2) $P/P_y = 0.3$

(3) $\sigma_{rc} = 0.3\,\sigma_y$

(4) $M_p = 1.11\ M_y$

(5) $M_A/M_y = 0.6$

(6) $M_B/M_y = -0.3$

(7) $\sigma_y = 33$ ksi, $E = 30 \times 10^3$ ksi

The nomographs used for this case are the ones constructed for equal end moments and restraints.

Solution: A horizontal line is drawn in the upper portion of the diagram for $M/M_y = -0.6$ (Fig. 11). The intersections with the M-θ curves are carried down to the lower portion of the nomograph to give distances L_A on each column deflection curve. Similarly the interactions with the horizontal line $M/M_y = +0.3$ are carried down to give the L_B distance. In the following table the values of L_A and L_B are added for each column deflection curve to give the length of column A-B. An arrow indicates the maximum value of L consistent with equilibrium.

θ_o	L_B/r	L_A/r	$L/r = \frac{L_A+L_B}{r}$
0.01	---	--	--
0.02	136	--	--
0.025	123	--	--
0.030	117	--	--
0.035	111	23	134
0.040	104.5	32	136.5 ←
0.050	85	30	115

$$(L/r)_{max} = 136.5 \text{ for } P/P_y = 0.3$$

(b) Unrestrained columns with moment at one end

Find the maximum end moment M_A of column A-B.

<u>Given</u>: (1) The pin-ended rolled steel, wide-flange column A-B of Fig. 12 with one end moment.

(2) $P/P_y = 0.3$

(3) $\sigma_{rc} = 0.3\,\sigma_y$

(4) $M_p = 1.11\,M_y$

(5) $L/r = 100$

(6) $\sigma_y = 33$ ksi, $E = 30 \times 10^3$ ksi

<u>Solution</u>: The nomographs used for this case are the ones constructed for one end pinned. A horizontal line is drawn in the lower portions of the appropriate nomograph at the specified slenderness ratio ($L/r = 100$). Intersections with the curves of the nomograph ($P/P_y = 0.3$) are carried up to the corresponding nomograph curves in upper portion. By connecting these points in the upper portion one obtains the relationships between the end moment and end rotation at point A for each equilibrium configuration. This is known as the M-θ curve for the given P/P_y.

From Fig. 12

$$(M/M_y)_{max} = -0.712 \text{ for } L/r = 100$$

$$\theta'_A = 0.0412$$

From Eq. (4)

$$\frac{M_A}{M_y} = - \frac{M}{M_y} \quad \text{for } f(\theta) = 0$$

$$(M/M_y)_{max} = 0.712 \text{ for } L/r = 100.$$

Moment-versus-end rotation curves for unrestrained columns, constructed by the method outlined above from the nomographs of Appendix VII-I are given in Appendix VII-II for strong axis bending and weak axis bending. These curves are included here because of their importance in the solution of problems in inelastic frame instability.[6,7]

IV. SUMMARY

This report is a continuation of Ref. 2, and both this report and Ref. 2 are the condensation of a Ph.D. dissertation.[1] Whereas Ref. 2 contains the theoretical background for the solutions of restrained column problems, the present paper presents the necessary curves and charts in order to solve practical problems. Because it would be required to have a set of charts for each type of material and cross sectional shape, and because each set of charts would require laborious numerical work, the contents of this report are of necessity limited to mild structural steel and to as-rolled wide-flange sections.

The most important part of the work presented herein is given in the Appendices. Appendix VII-I gives nomographs which depict the relationships between the slope, the bending moment and the location of any point on a column deflection curve for strong axis bending.

There are two types of nomographs: One type contains charts for symmetrically arranged column curves (that is, equal end moments and equal end restraints), and the other type is for the case where one end of the column is free of end moment and end restraint. Each nomograph is constructed for a specific axial force. One set of curves each is provided for $P = 0.12\ P_y$, $0.2\ P_y$, $0.3\ P_y$, $0.4\ P_y$, and $0.6\ P_y$.

The use of these nomographs is illustrated in the report by examples of solved problems.

The critical combinations of loading and geometry are obtained for the following problems:

a) Beam-columns having equal end moments and end restraints.

b) Beam-columns with one end pinned.

c) Pinned-end beam-columns having any combination of end moment.

It is shown that rapid solutions can be found by graphical procedures, using the nomographs.

Appendix VII-II gives moment-versus-end rotation curves for pinned-end columns subjected to axial force and (a) equal end moments causing single curvature deformation, (b) end moment only at one end. Curves are given for both strong and weak axis bending. These curves are of importance because they show the complete history of a column, especially in the range where large inelastic rotations may exist. The moment-rotation curves are of importance in solving problems in frame stability.[6,7]

V. ACKNOWLEDGMENTS

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VI. NOMENCLATURE

A = Cross sectional area of a column (in^2)

P = Compressive load (lbs)

E = Young's Modulus (psi)

P_y = $\sigma_y A$ (lbs)

I = Moment of inertia about axis of bending (in^4)

M = Bending moment (lb-in)

M_p = Plastic bending moment of a cross section (lb -in)

M_y = Bending moment of a cross section at initial yield

L = Column length

e_A, e_B = Eccentricities with which a load is applied to a structure (in)

d = Depth of a column cross section (in)

r = Radius of gyration about axis of bending (in)

s = Length of restraining span (in)

y_m = Maximum amplitude of a column deflection curve (in)

y, y_A, y_B, y_n = Deflections of points on a column deflection curve (in)

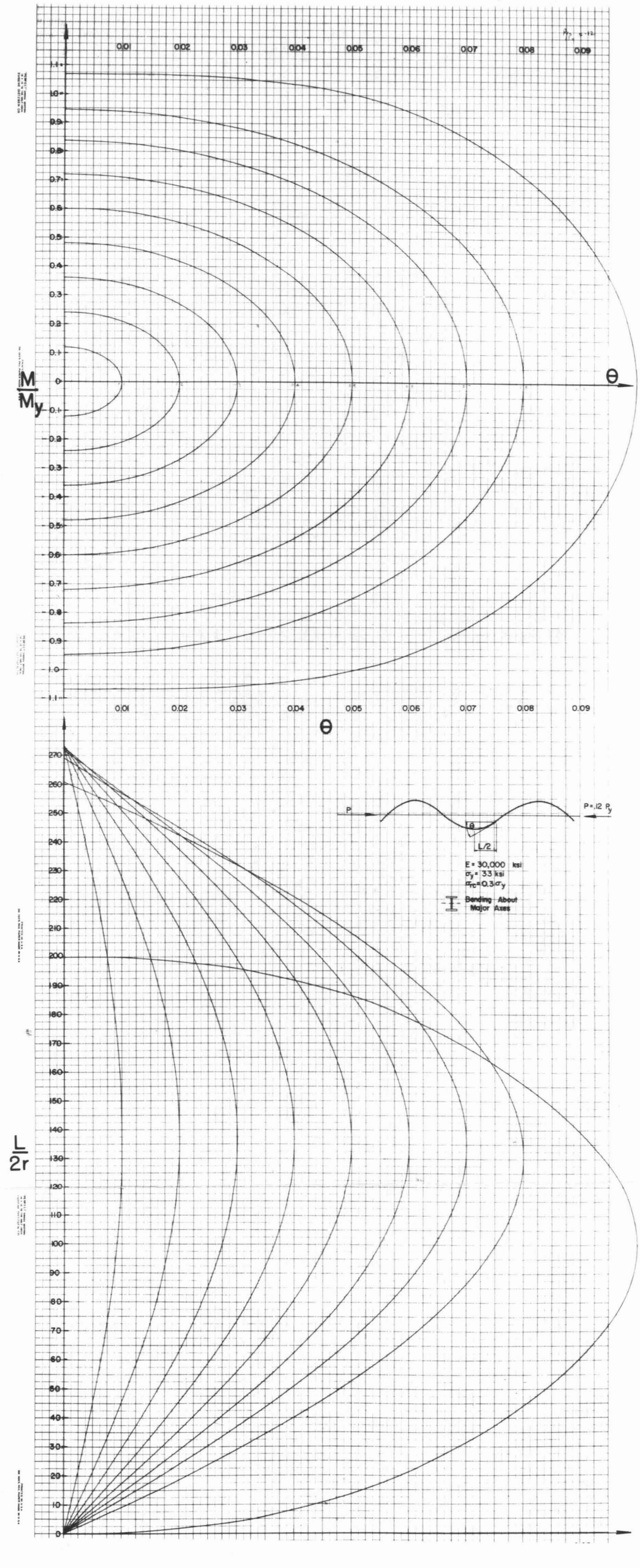
σ_{rc} = Maximum residual compressive stress (psi)

$f_A(\theta), f_B(\theta)$ =Functions giving restraining moments at column ends (lb.in./radian)

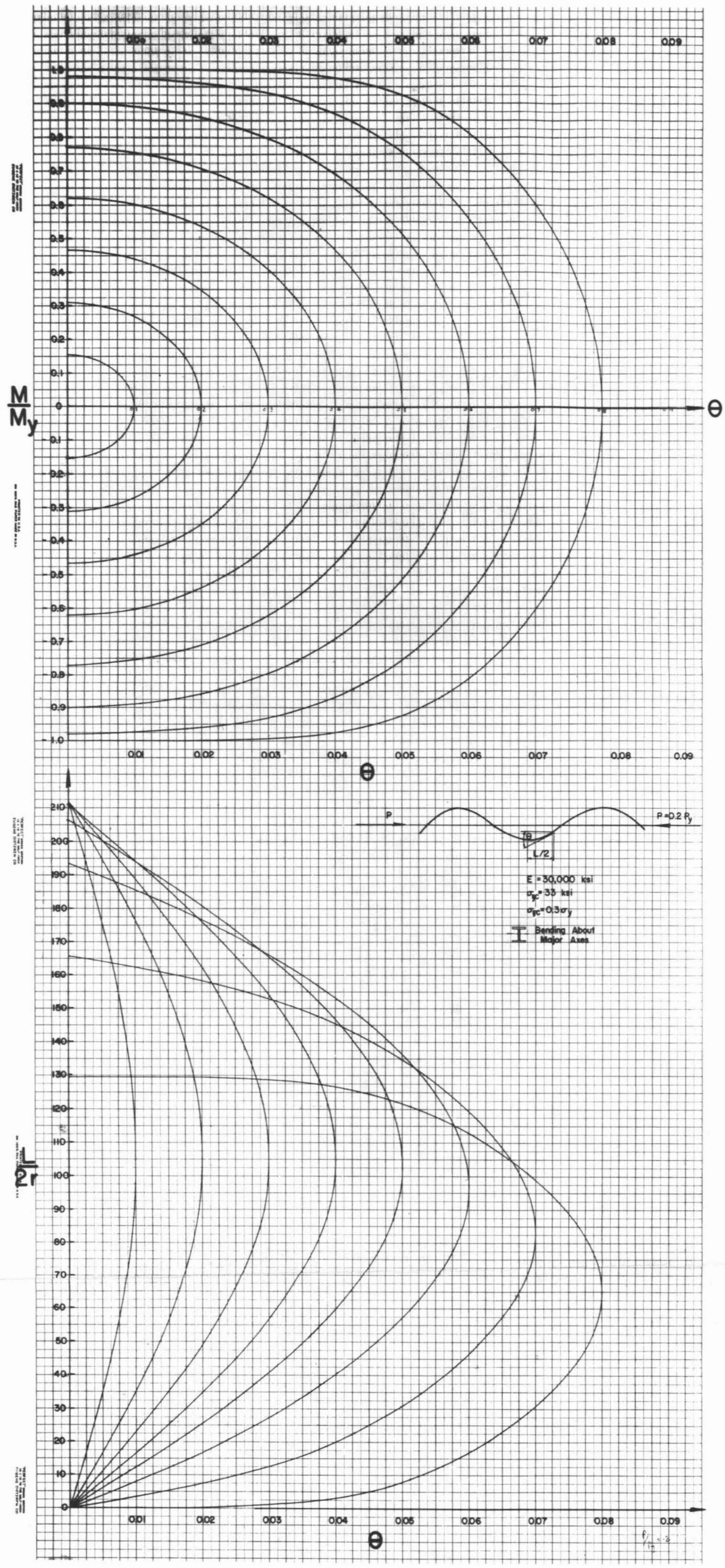
278.5

VII. APPENDICES

VII.1 M-Θ-L NOMOGRAPHS



0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
M/My
θ
L/2r
P
P = .12 Py
θ
L/2
E = 30,000 ksi
σy = 33 ksi
σrc = 0.3 σy
Bending About Major Axes



M/My
Θ
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
P
P = 0.2 P_y
θ
L/2
E = 30,000 ksi
σ_{yc} = 33 ksi
σ_{rc} = 0.3 σ_y
Bending About Major Axes
L/2r
210
200
190
180
170
160
150
140
130
120
110
100
90
80
70
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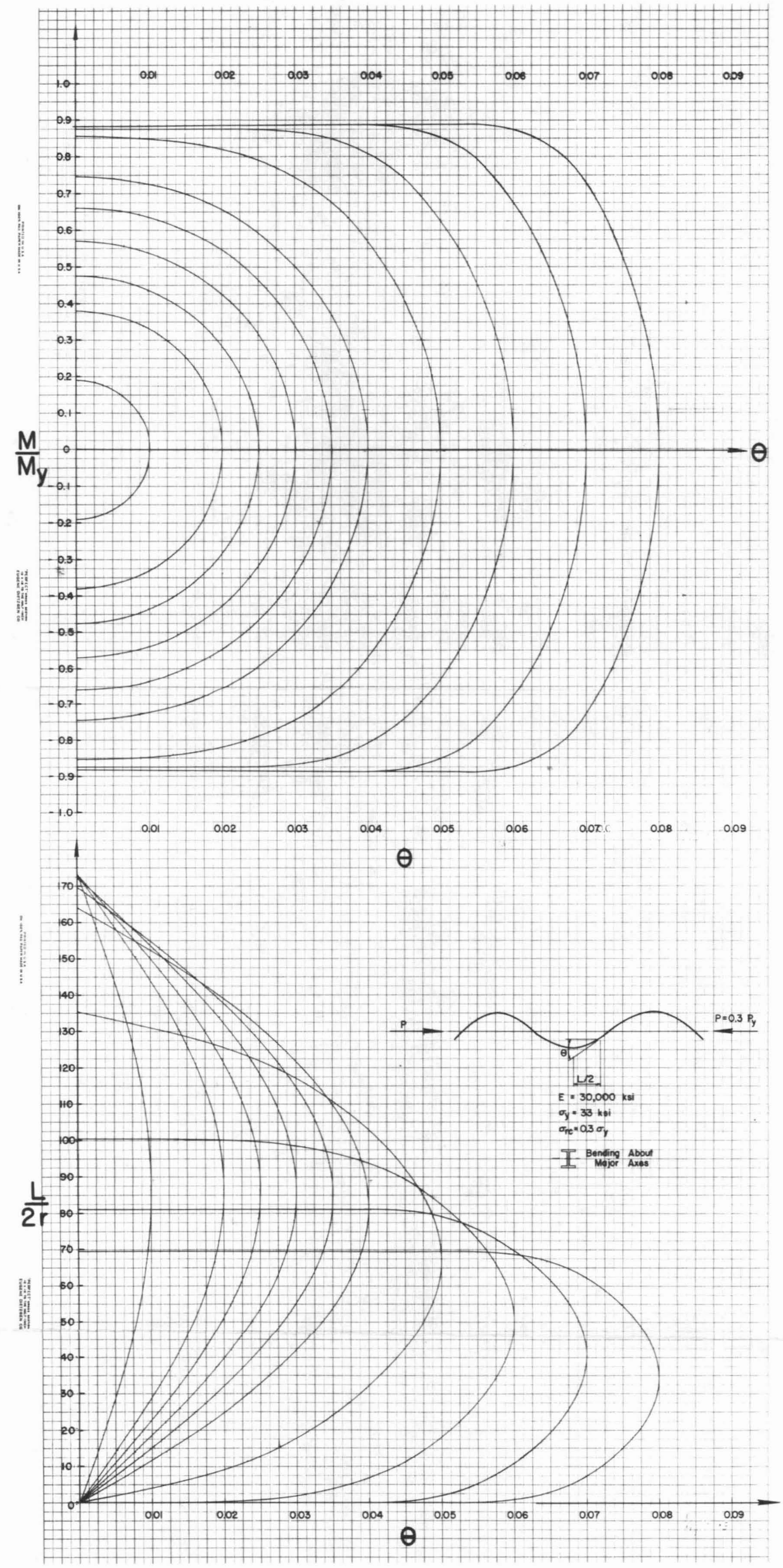
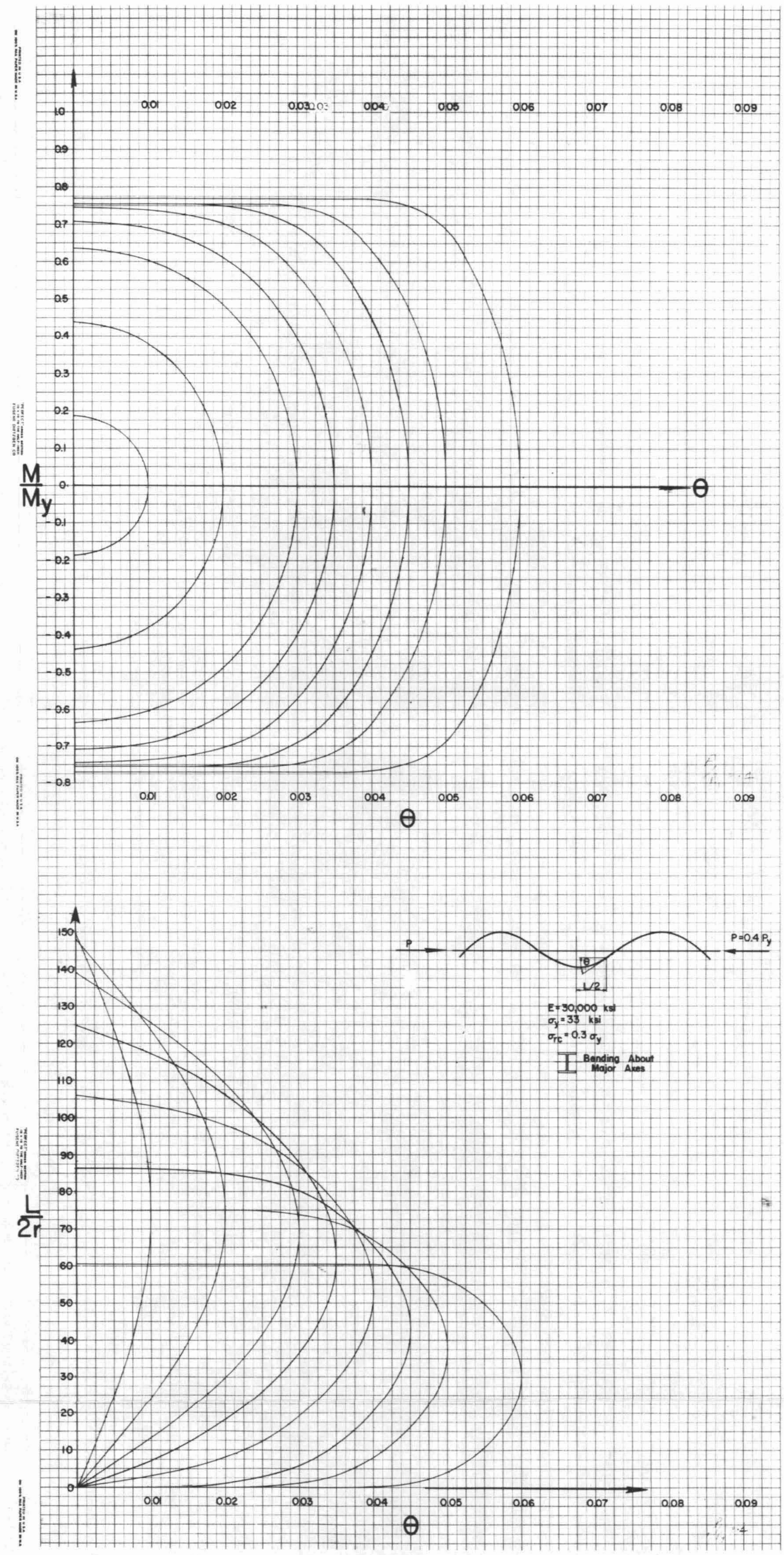
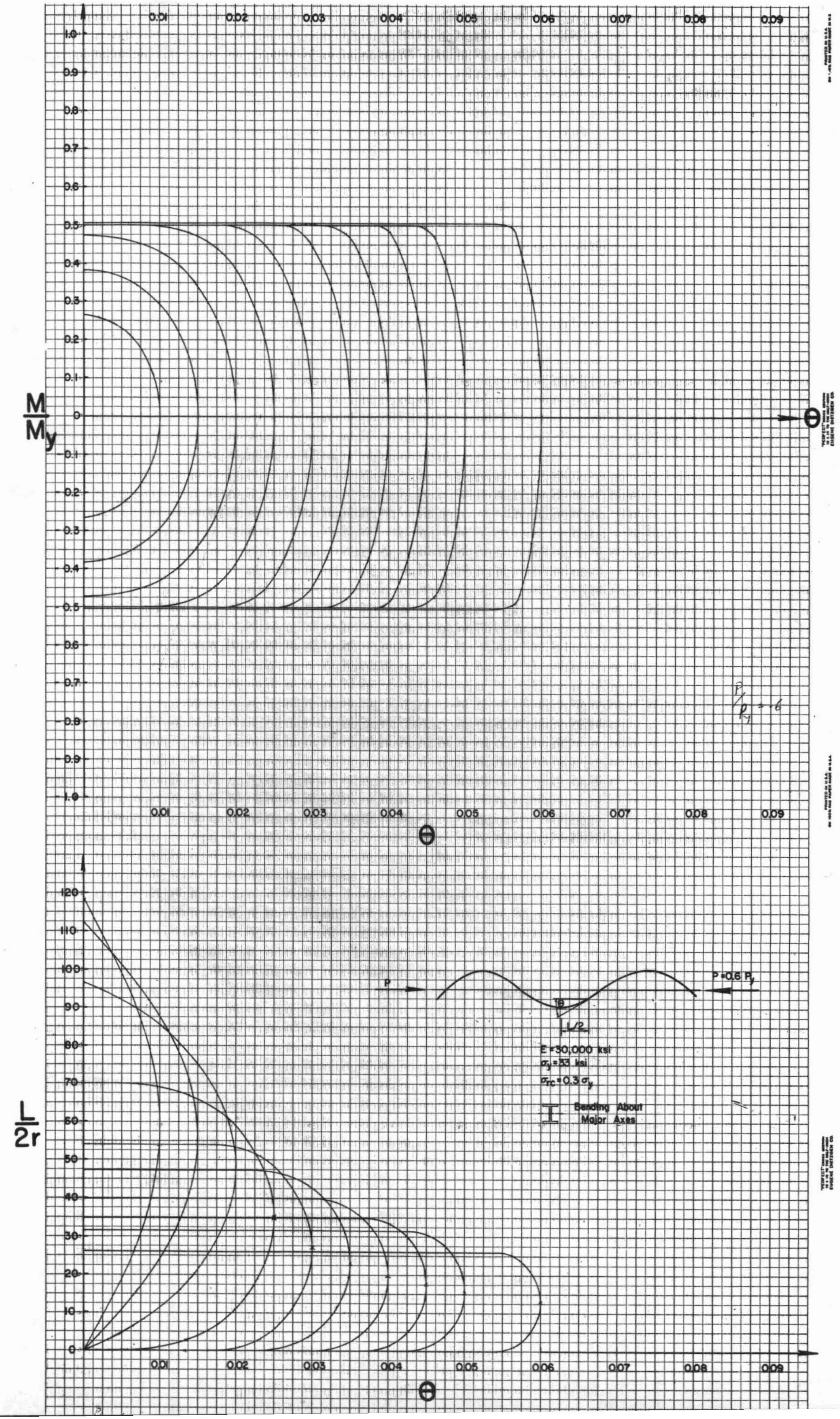
50
40
30
20
10



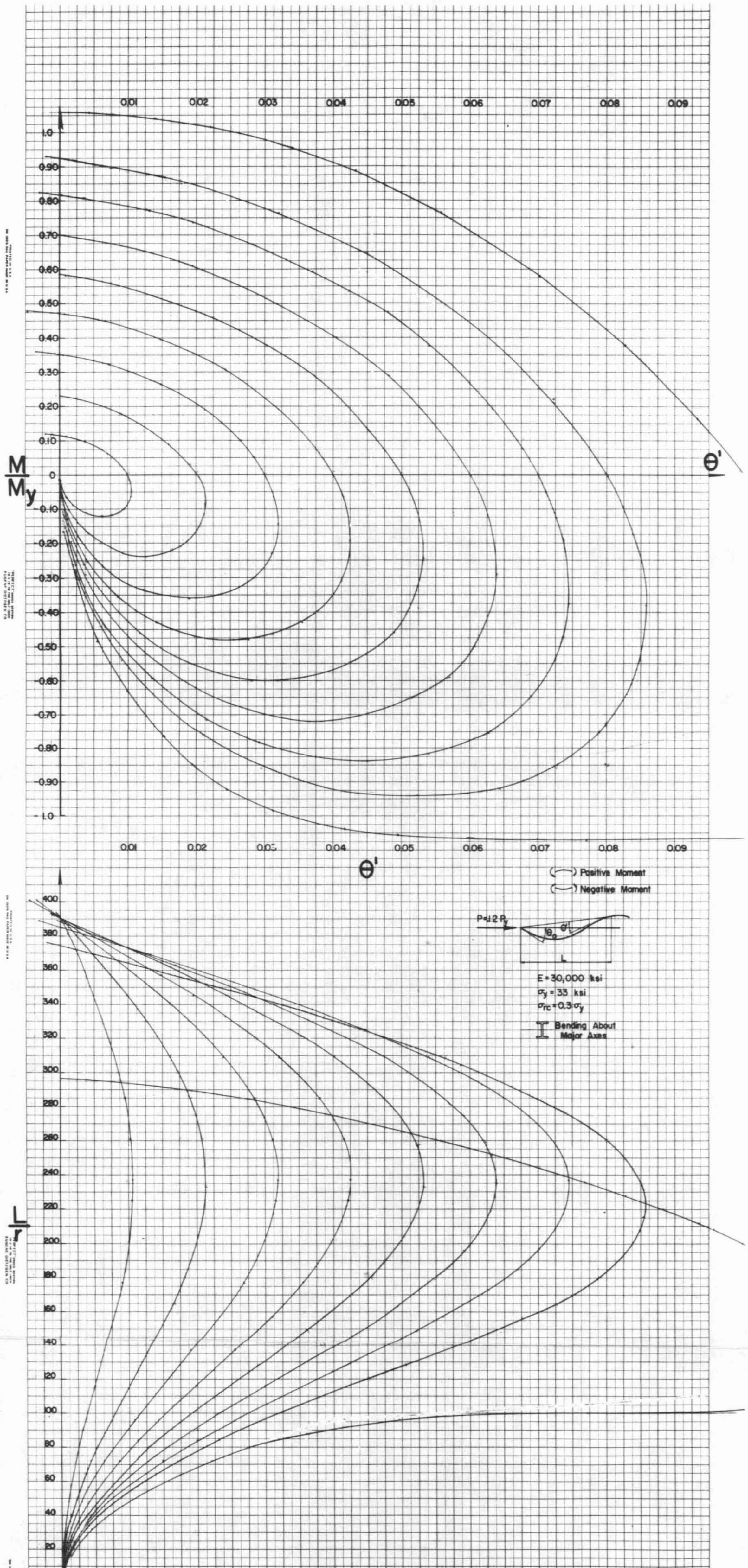
M/My
Θ
L/2r
P
P=0.3 Py
θ
L/2
E = 30,000 ksi
σy = 33 ksi
σrc = 0.3 σy
Bending About Major Axes

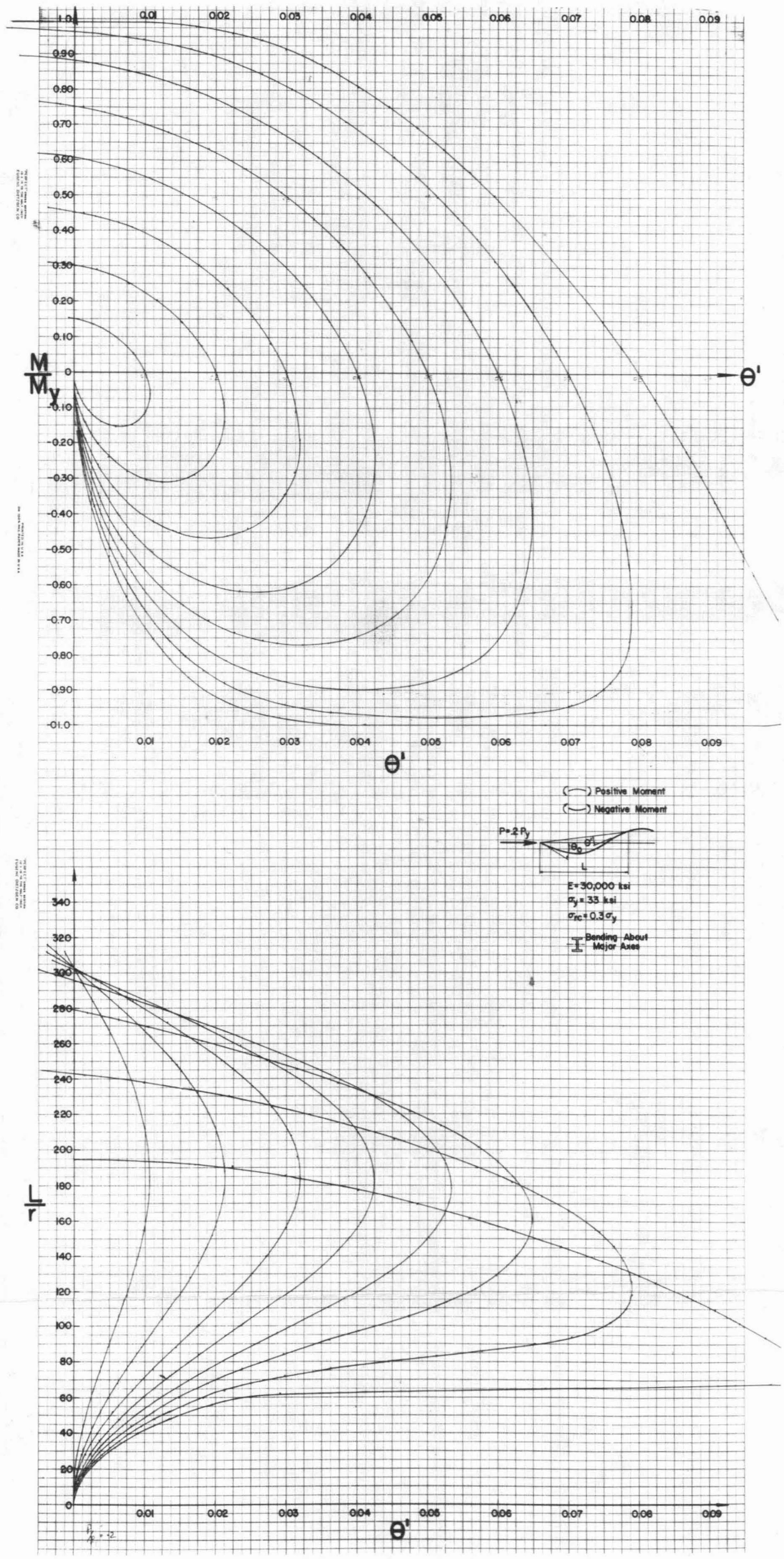


M/My
θ
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
L/2r
P
P=0.4 Py
θ
L/2
E=30,000 ksi
σy=33 ksi
σrc=0.3 σy
Bending About Major Axes

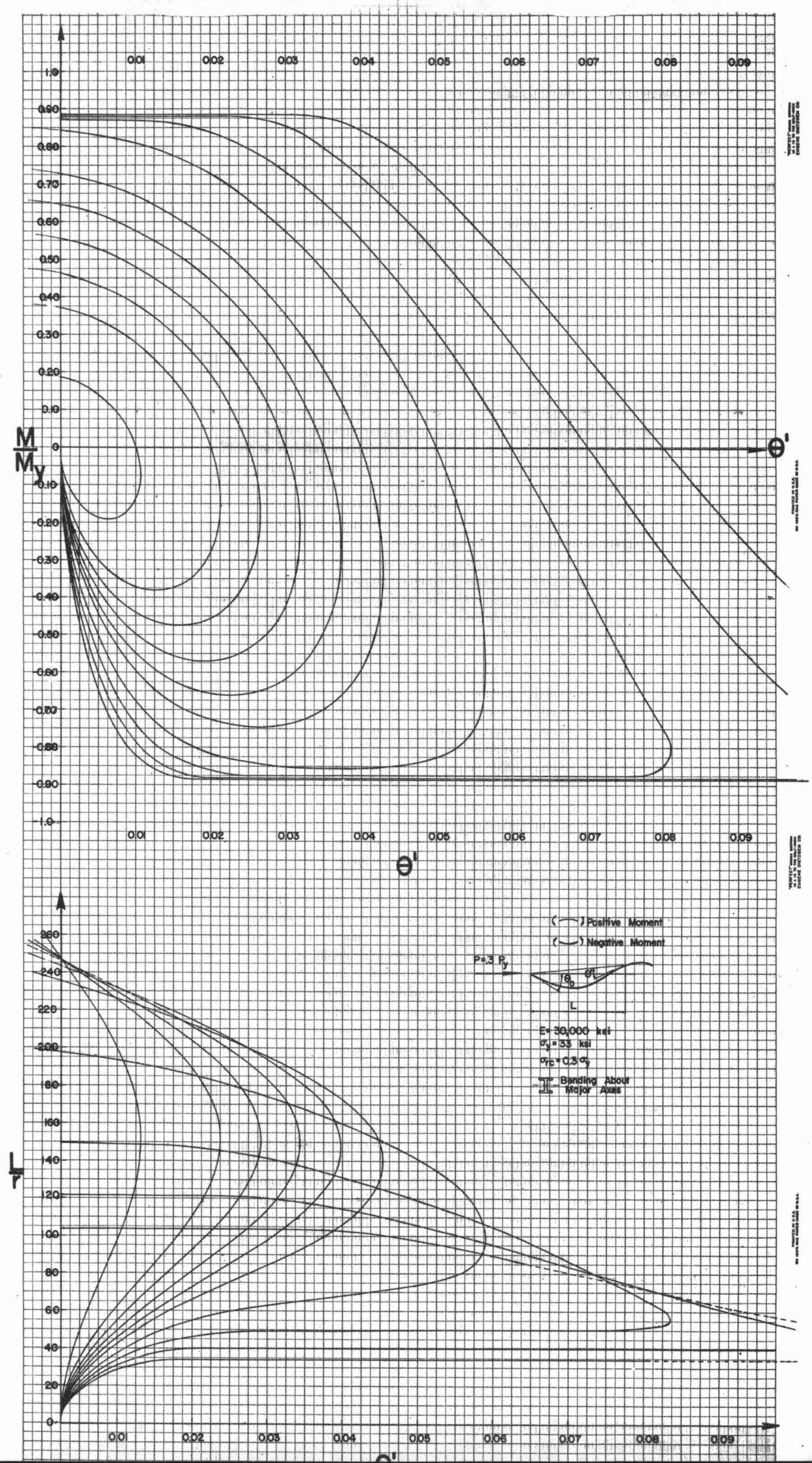


M/My
Θ
L/2r
P = 0.6 P_y
P
θ
L/2
E = 30,000 ksi
σ_y = 33 ksi
σ_{rc} = 0.3 σ_y
Bending About Major Axes
P/Py = .6

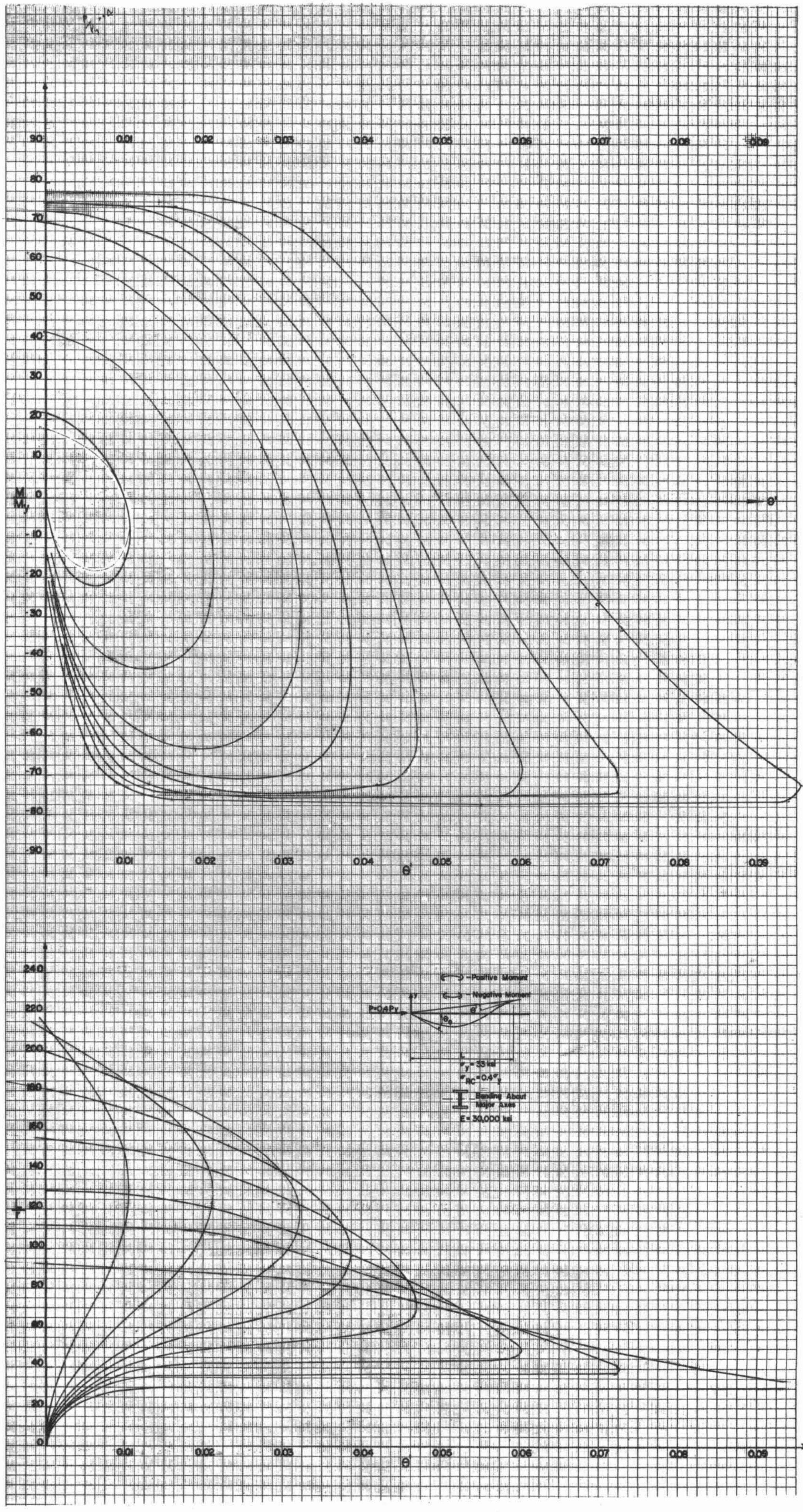

M/My
θ'
L/r
Positive Moment
Negative Moment
P=1.2 Py
θo
θ'
L
E = 30,000 ksi
σy = 33 ksi
σrc = 0.3 σy
Bending About Major Axes



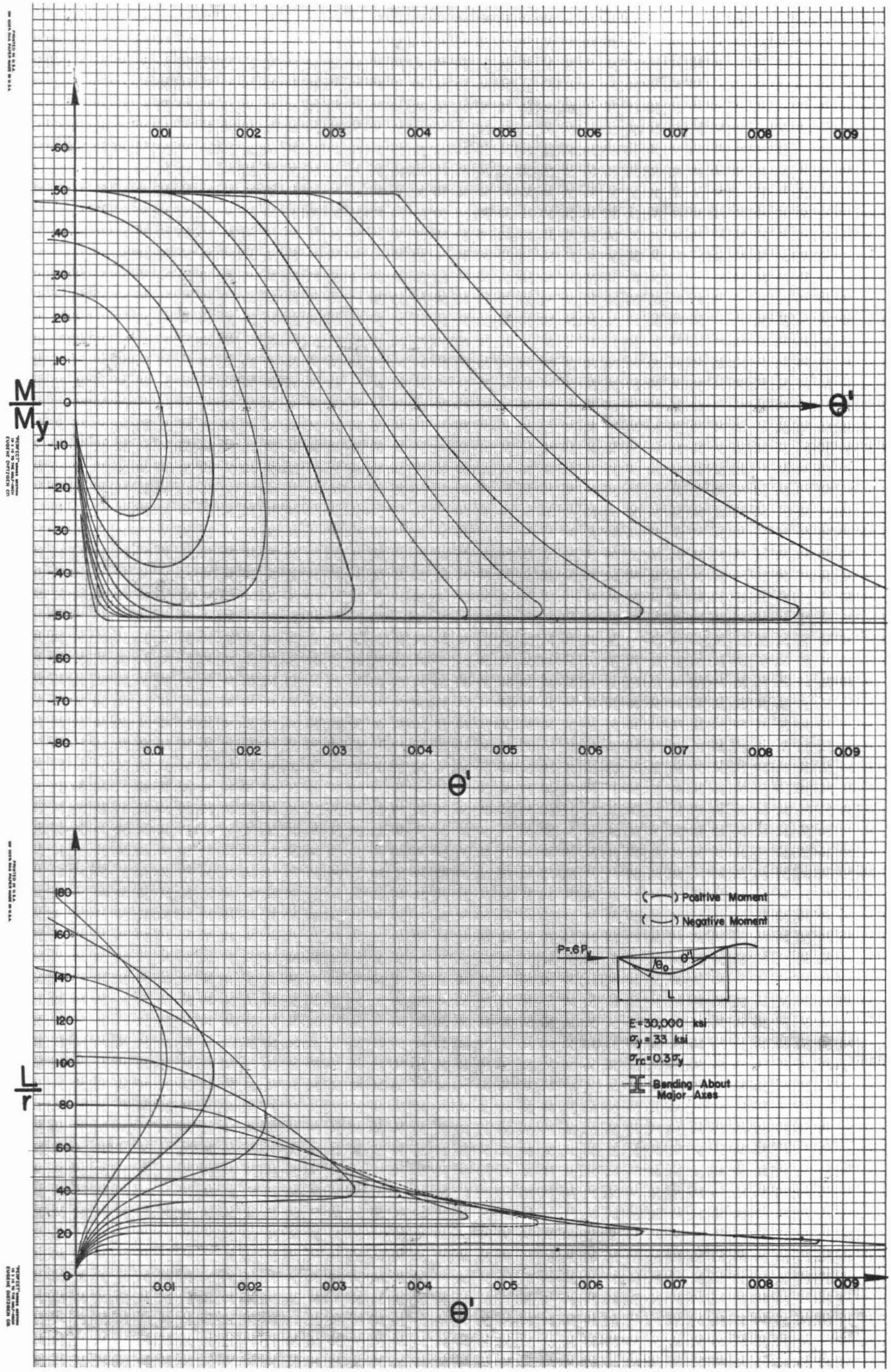
M/My
θ'
L/r
( ) Positive Moment
( ) Negative Moment
P=.2 Py
L
E=30,000 ksi
σy = 33 ksi
σrc = 0.3σy
Bending About Major Axes



M/My
Θ'
0.01 0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09
1.0 0.90 0.80 0.70 0.60 0.50 0.40 0.30 0.20 0.10 0
-0.10 -0.20 -0.30 -0.40 -0.50 -0.60 -0.70 -0.80 -0.90 -1.0
L/r
260 240 220 200 180 160 140 120 100 80 60 40 20 0
Positive Moment
Negative Moment
P=.3 Py
θo
θ'L
L
E= 30,000 ksi
σy = 33 ksi
σrc = 0.3 σy
Bending About Major Axes

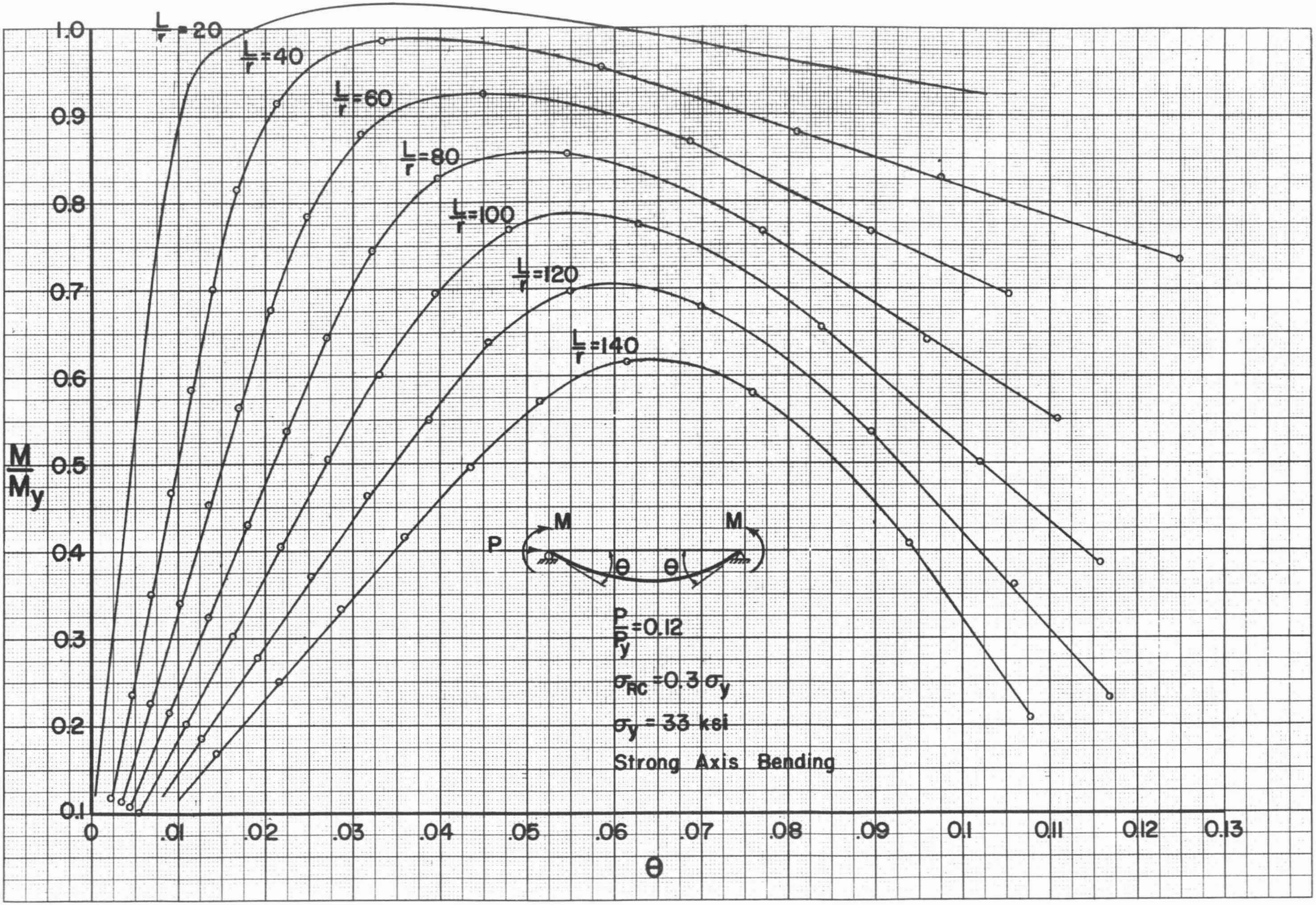


M/My
θ'
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
90
80
70
60
50
40
30
20
10
0
-10
-20
-30
-40
-50
-60
-70
-80
-90
L/r
240
220
200
180
160
140
120
100
80
60
40
20
- Positive Moment
- Negative Moment
P=0.4Py
y
θ'
θ₀
L
σy = 33 ksi
σRC = 0.4σy
Bending About Major Axes
E = 30,000 ksi

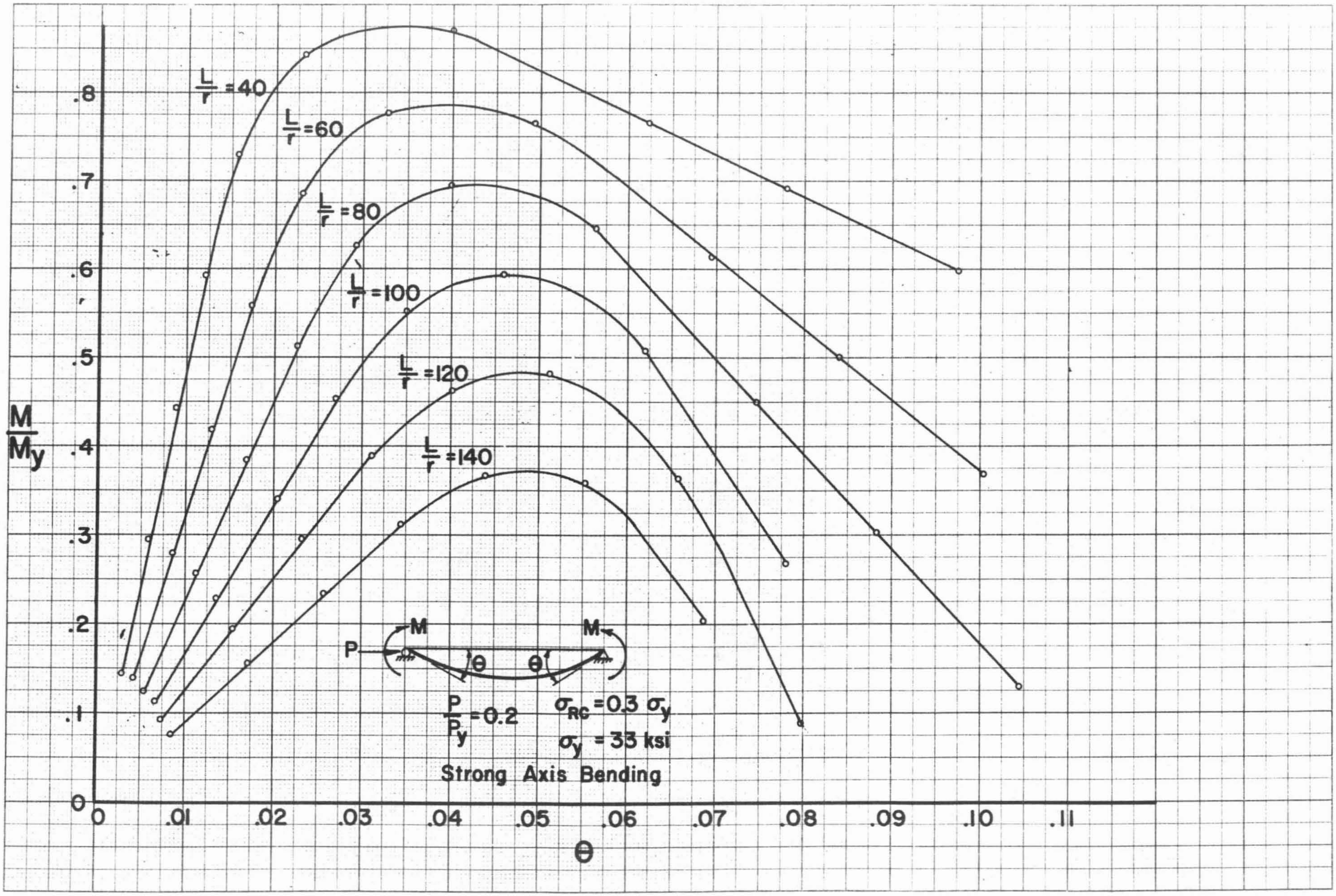


M/My
Θ'
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
.60
.50
.40
.30
.20
.10
0
-.10
-.20
-.30
-.40
-.50
-.60
-.70
-.80
L/r
180
160
140
120
100
80
60
40
20
0
( ) Positive Moment
( ) Negative Moment
P=.6Py
θo
θ'
L
E=30,000 ksi
σy = 33 ksi
σrc=0.3σy
Bending About Major Axes

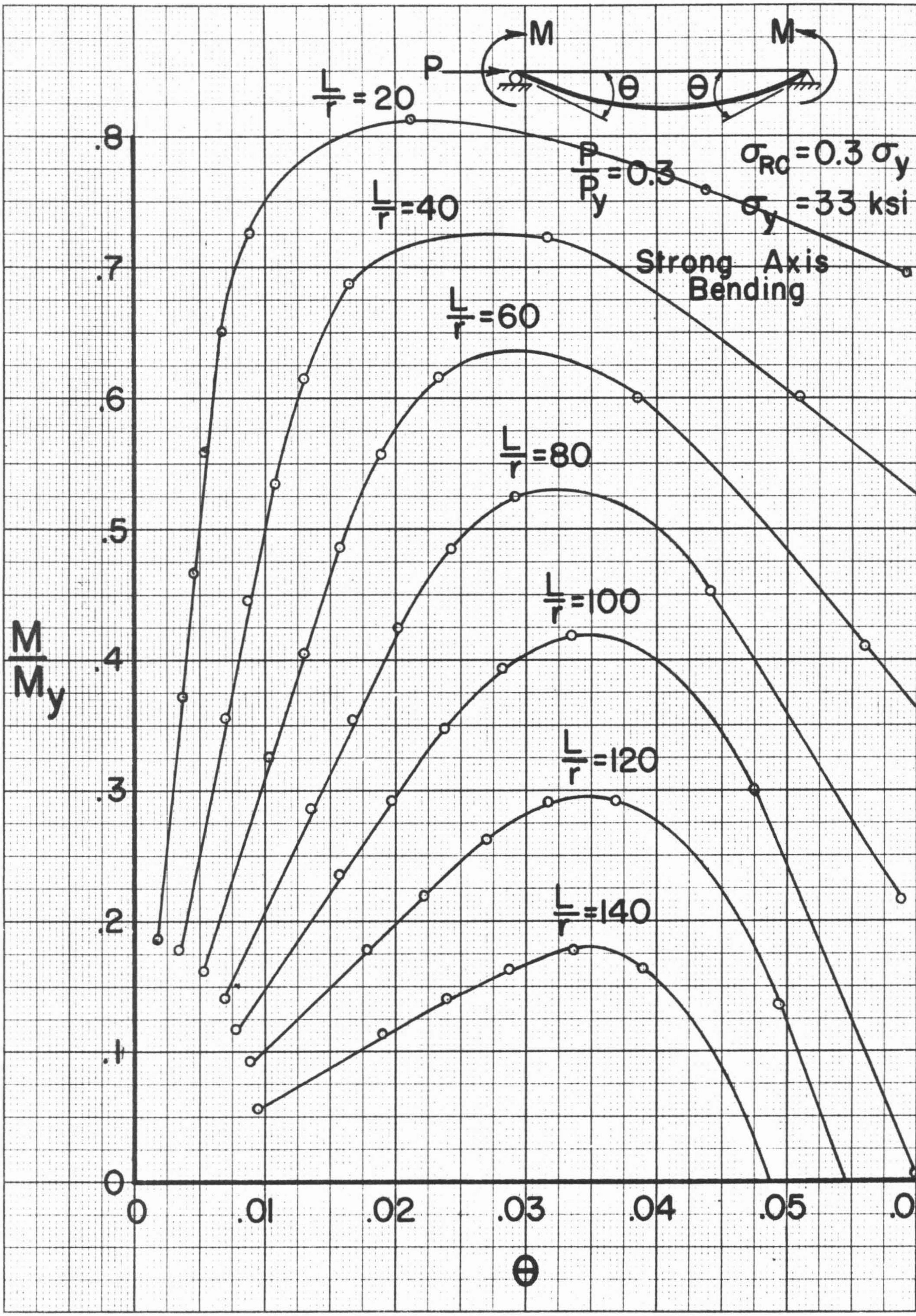
VII.2 M-Θ CURVES



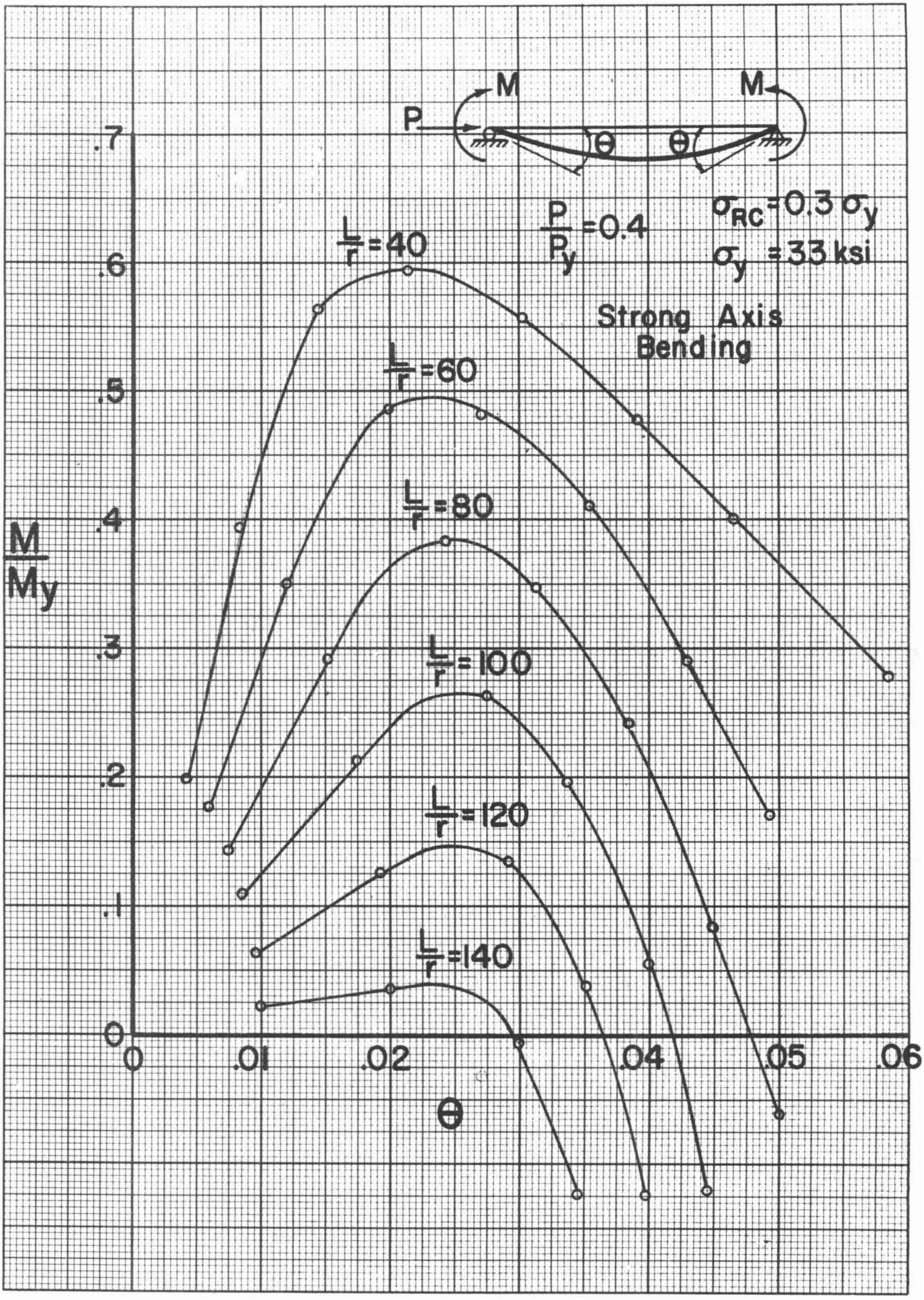
L/r = 20
L/r = 40
L/r = 60
L/r = 80
L/r = 100
L/r = 120
L/r = 140
M/My
θ
P
M
M
θ
θ
P/Py = 0.12
σRC = 0.3 σy
σy = 33 ksi
Strong Axis Bending



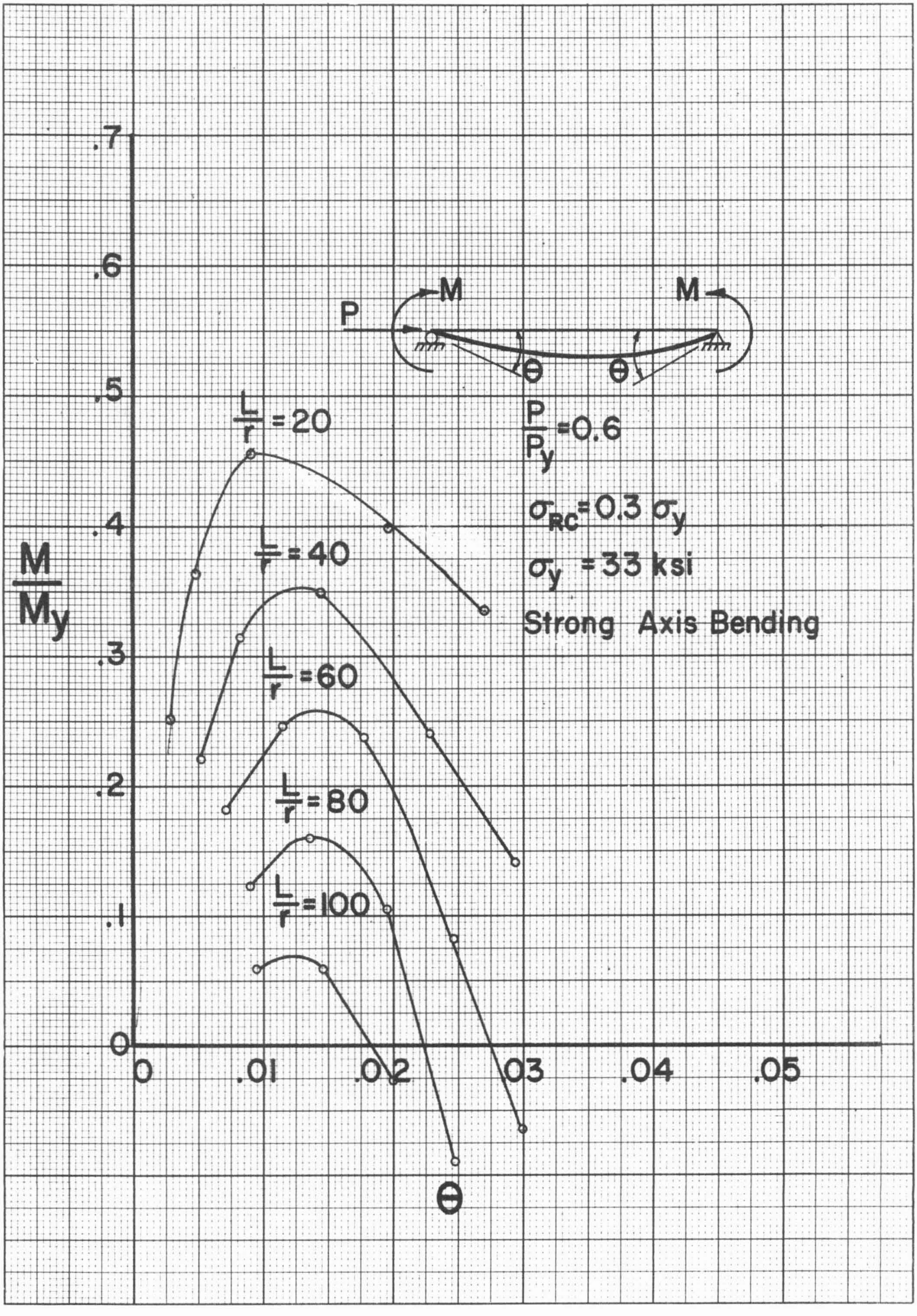
L/r = 40
L/r = 60
L/r = 80
L/r = 100
L/r = 120
L/r = 140
M/My
θ
P
M
M
θ
θ
P/Py = 0.2
σRC = 0.3 σy
σy = 33 ksi
Strong Axis Bending
0
.1
.2
.3
.4
.5
.6
.7
.8
0
.01
.02
.03
.04
.05
.06
.07
.08
.09
.10
.11



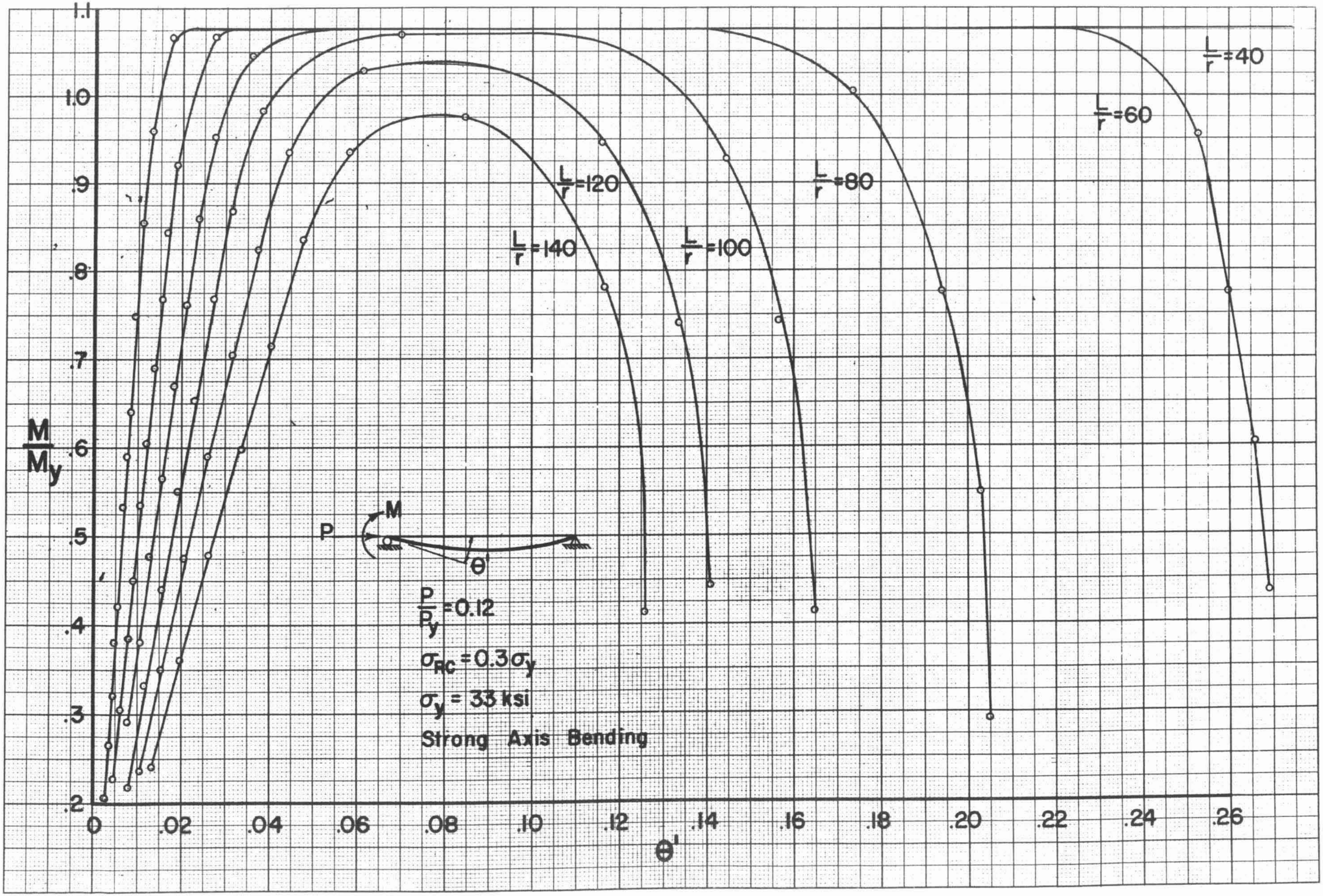
M
P
M
θ
θ
L/r = 20
L/r = 40
L/r = 60
L/r = 80
L/r = 100
L/r = 120
L/r = 140
P/Py = 0.3
σRC = 0.3 σy
σy = 33 ksi
Strong Axis Bending
M/My
.8
.7
.6
.5
.4
.3
.2
.1
0
0
.01
.02
.03
.04
.05
.06
θ



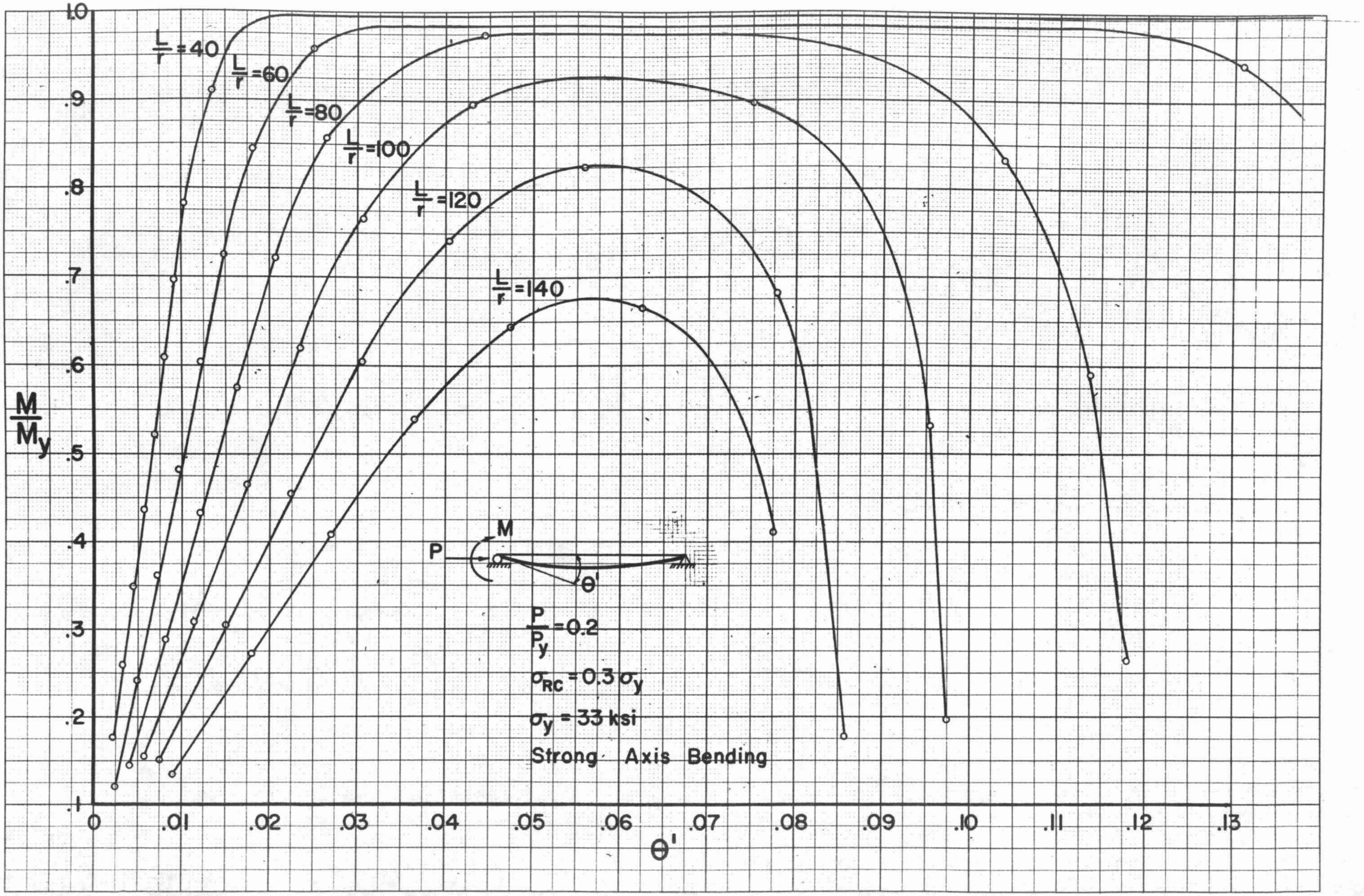
M
P
Θ
$\frac{P}{P_y}$ = 0.4
σ_{RC} = 0.3 σ_y
σ_y = 33 ksi
Strong Axis Bending
$\frac{L}{r}$ = 40
$\frac{L}{r}$ = 60
$\frac{L}{r}$ = 80
$\frac{L}{r}$ = 100
$\frac{L}{r}$ = 120
$\frac{L}{r}$ = 140
$\frac{M}{M_y}$
.7
.6
.5
.4
.3
.2
.1
0
0
.01
.02
.04
.05
.06



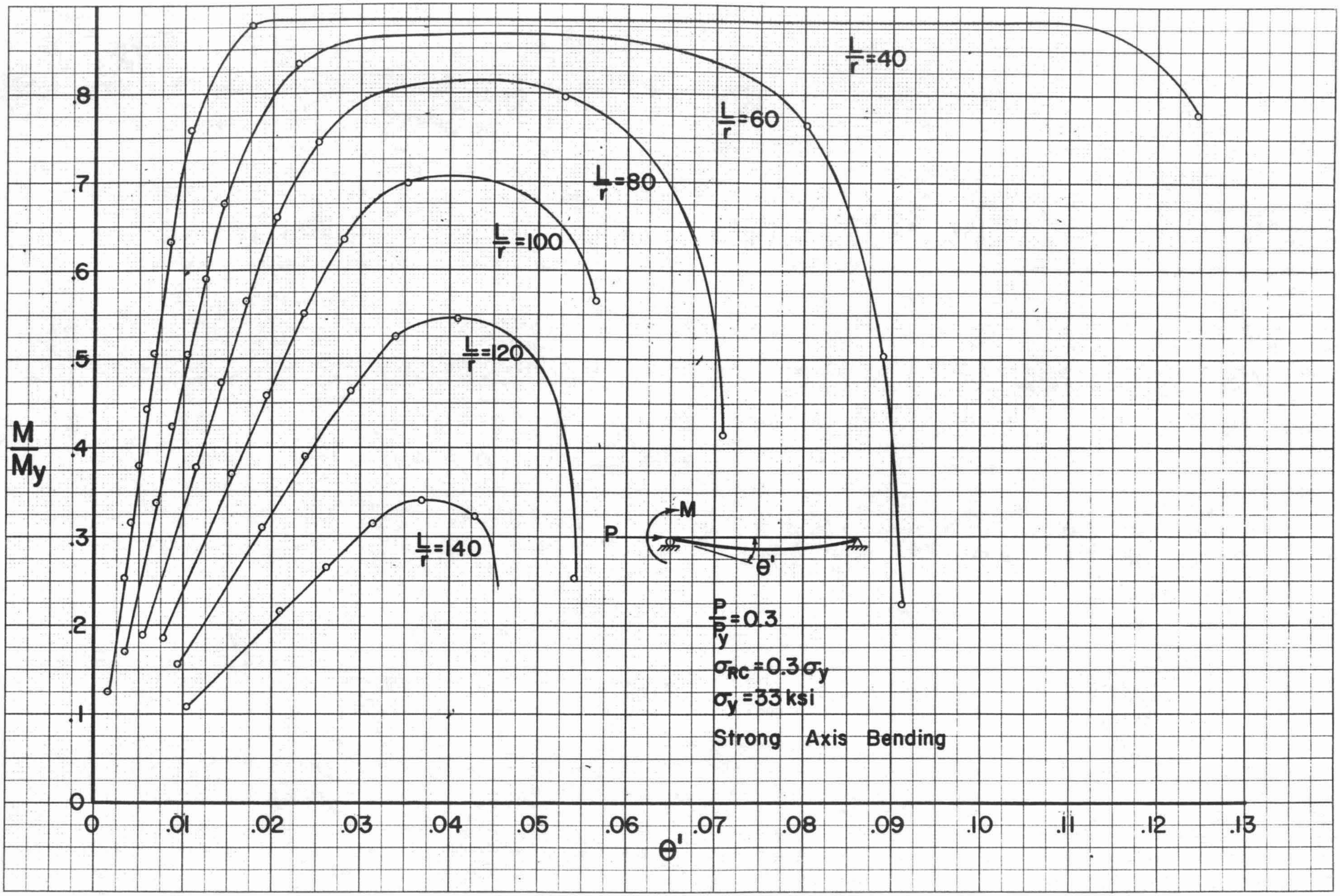
.7
.6
.5
.4
.3
.2
.1
0
M / My
0
.01
.02
.03
.04
.05
Θ
P
M
M
Θ
Θ
L/r = 20
L/r = 40
L/r = 60
L/r = 80
L/r = 100
P/Py = 0.6
σRC = 0.3 σy
σy = 33 ksi
Strong Axis Bending



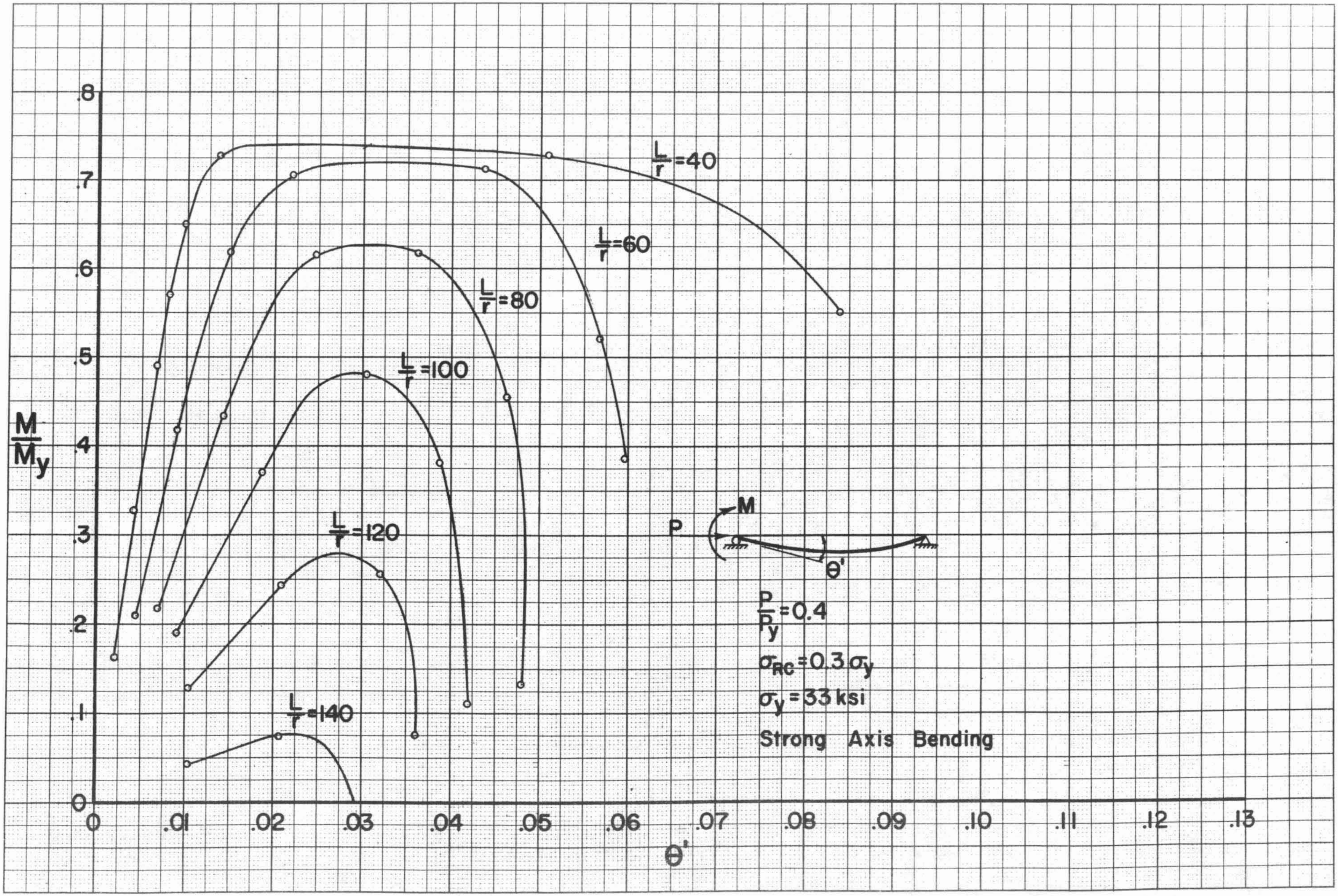
$\frac{M}{M_y}$
θ'
$\frac{L}{r}=40$
$\frac{L}{r}=60$
$\frac{L}{r}=80$
$\frac{L}{r}=100$
$\frac{L}{r}=120$
$\frac{L}{r}=140$
P
M
θ'
$\frac{P}{P_y}=0.12$
$\sigma_{RC}=0.3\sigma_y$
σ_y = 33 ksi
Strong Axis Bending



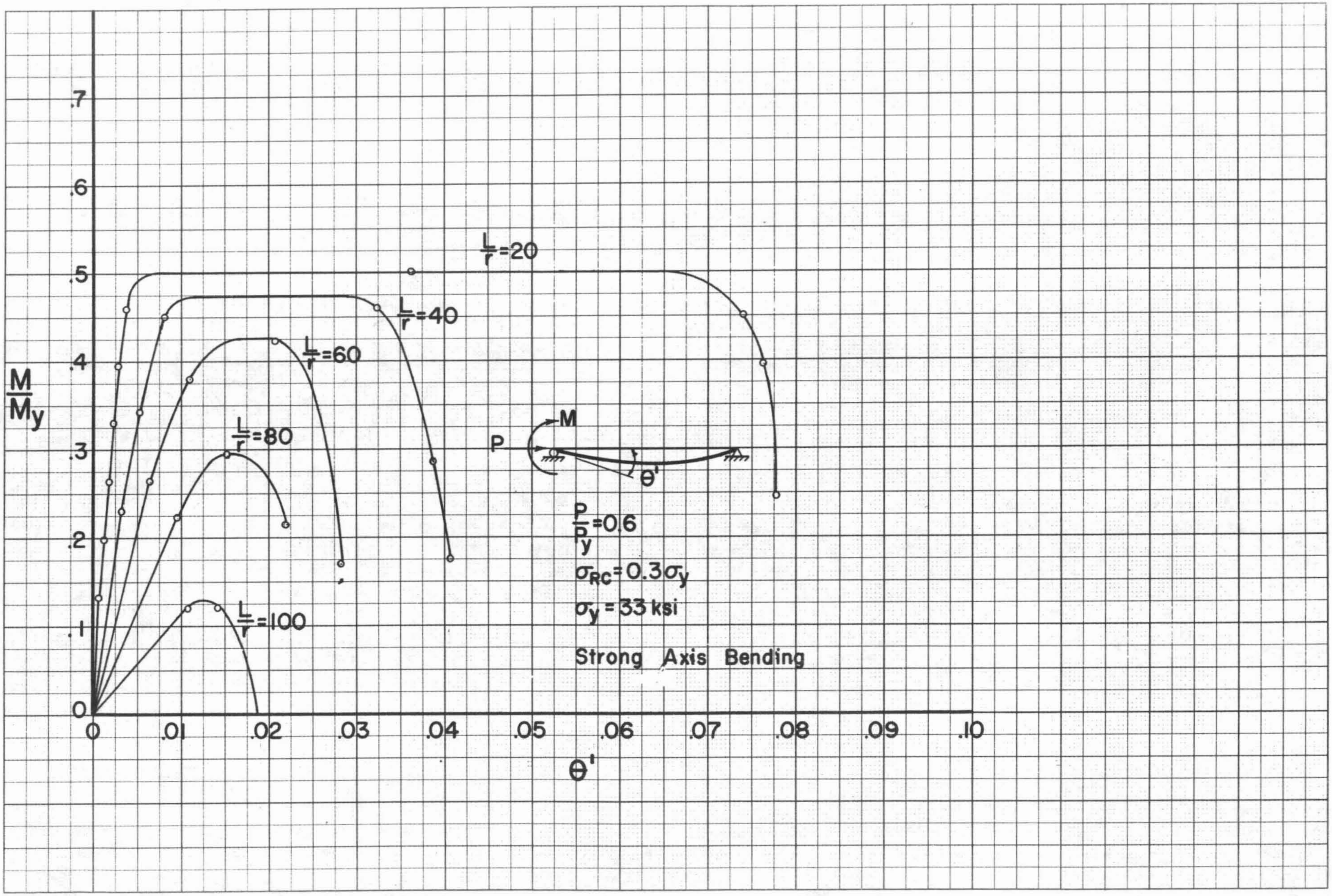
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
$\frac{M}{M_y}$
$\frac{L}{r}=40$
$\frac{L}{r}=60$
$\frac{L}{r}=80$
$\frac{L}{r}=100$
$\frac{L}{r}=120$
$\frac{L}{r}=140$
M
P
θ'
$\frac{P}{P_y}=0.2$
$\sigma_{RC}=0.3\,\sigma_y$
$\sigma_y=33$ ksi
Strong Axis Bending
0
.01
.02
.03
.04
.05
.06
.07
.08
.09
.10
.11
.12
.13
θ'



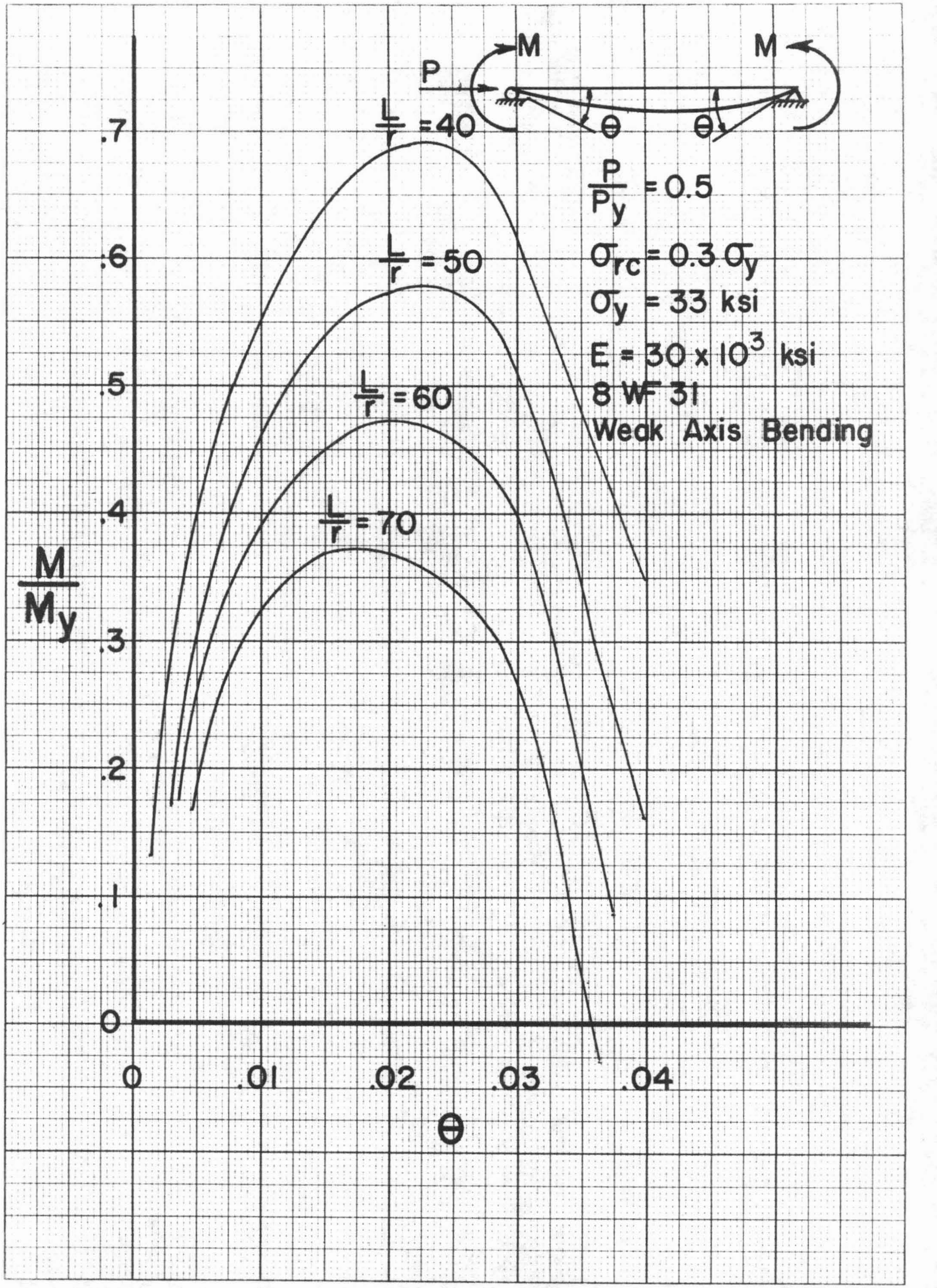
$\frac{L}{r}=40$
$\frac{L}{r}=60$
$\frac{L}{r}=80$
$\frac{L}{r}=100$
$\frac{L}{r}=120$
$\frac{L}{r}=140$
$\frac{M}{M_y}$
θ'
P
M
θ'
$\frac{P}{P_y}=0.3$
$\sigma_{RC}=0.3\sigma_y$
$\sigma_y=33$ ksi
Strong Axis Bending
0
.1
.2
.3
.4
.5
.6
.7
.8
0
.01
.02
.03
.04
.05
.06
.07
.08
.09
.10
.11
.12
.13



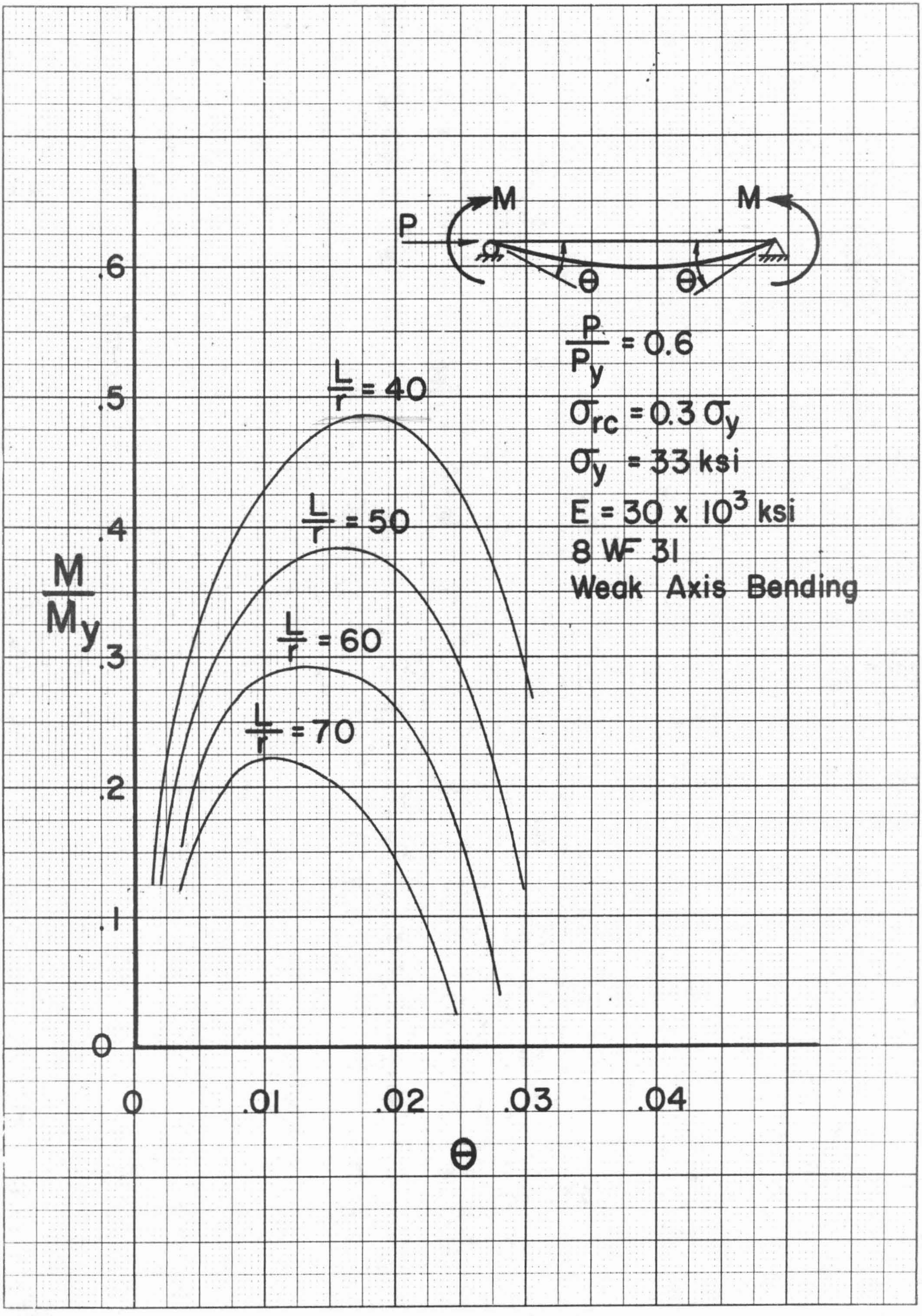
M/My
.8
.7
.6
.5
.4
.3
.2
.1
0
0
.01
.02
.03
.04
.05
.06
.07
.08
.09
.10
.11
.12
.13
θ'
L/r = 40
L/r = 60
L/r = 80
L/r = 100
L/r = 120
L/r = 140
M
P
θ'
P/Py = 0.4
σRC = 0.3 σy
σy = 33 ksi
Strong Axis Bending



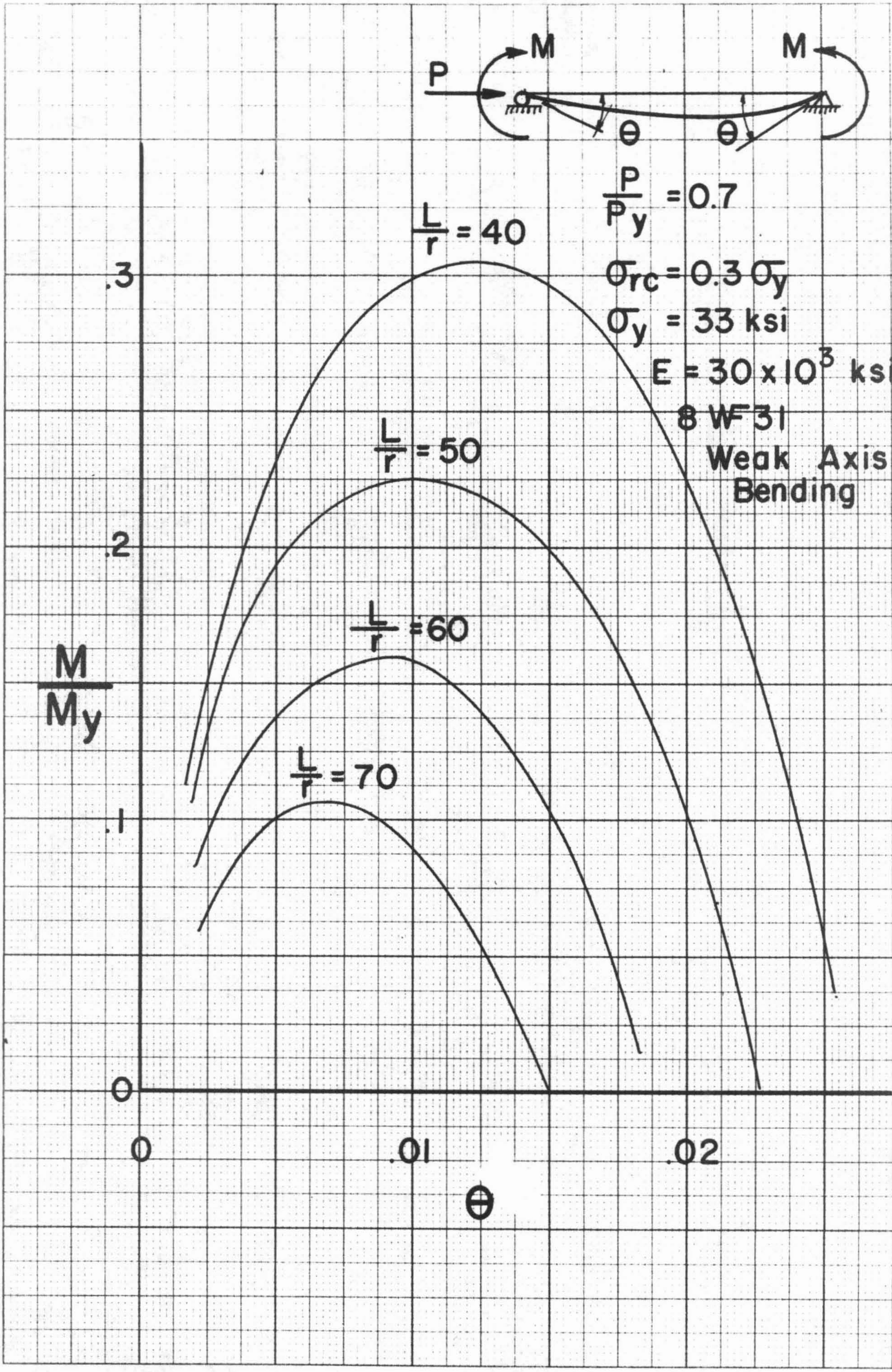
$\frac{M}{M_y}$
.7
.6
.5
.4
.3
.2
.1
0
$\frac{L}{r}=20$
$\frac{L}{r}=40$
$\frac{L}{r}=60$
$\frac{L}{r}=80$
$\frac{L}{r}=100$
M
P
θ'
$\frac{P}{P_y}=0.6$
$\sigma_{RC}=0.3\sigma_y$
$\sigma_y=33$ ksi
Strong Axis Bending
0
.01
.02
.03
.04
.05
.06
.07
.08
.09
.10
θ'



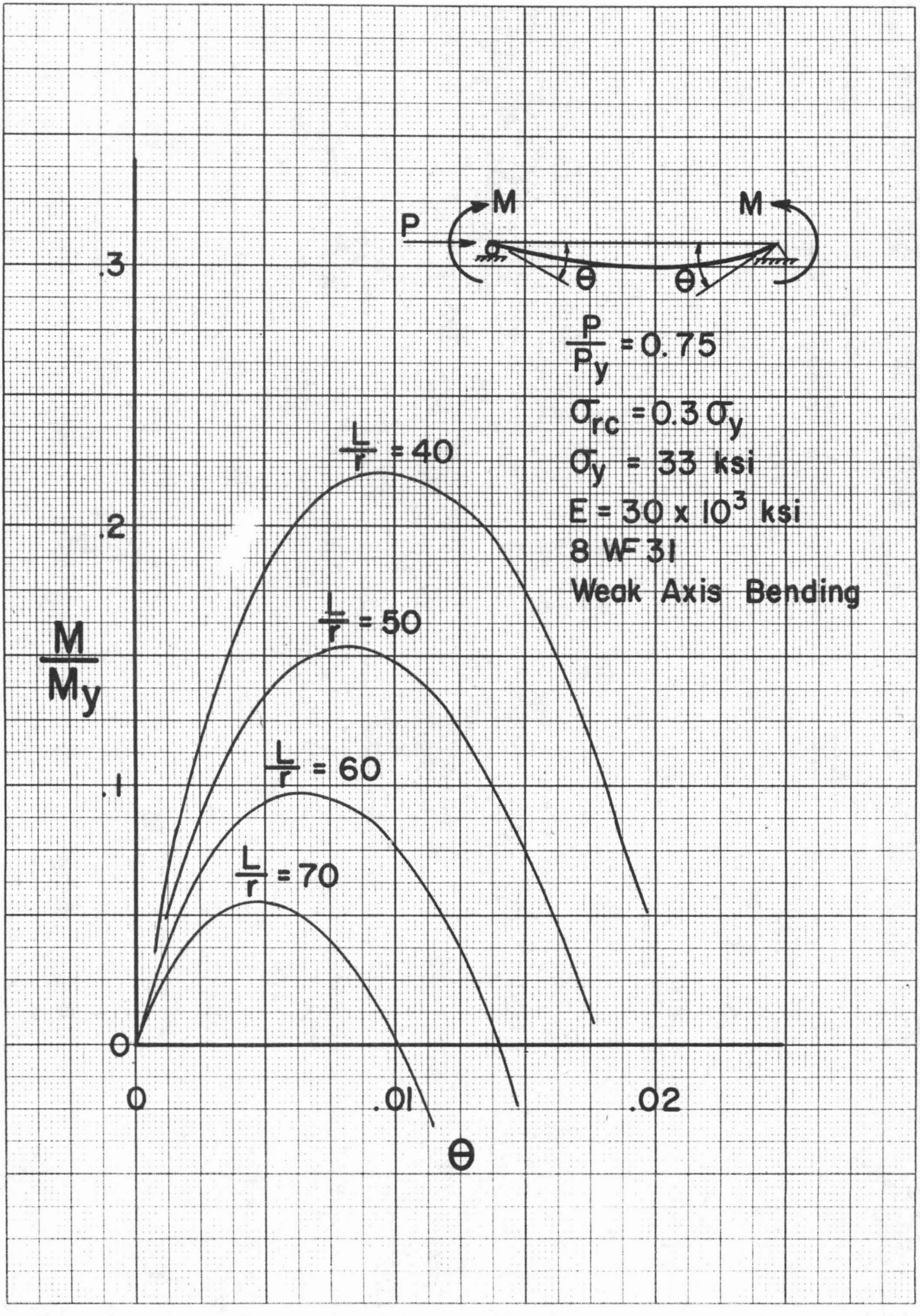
M
M
P
θ
θ
L/r = 40
L/r = 50
L/r = 60
L/r = 70
P/Py = 0.5
σrc = 0.3 σy
σy = 33 ksi
E = 30 x 10^3 ksi
8 WF 31
Weak Axis Bending
M/My
.7
.6
.5
.4
.3
.2
.1
0
0
.01
.02
.03
.04
θ



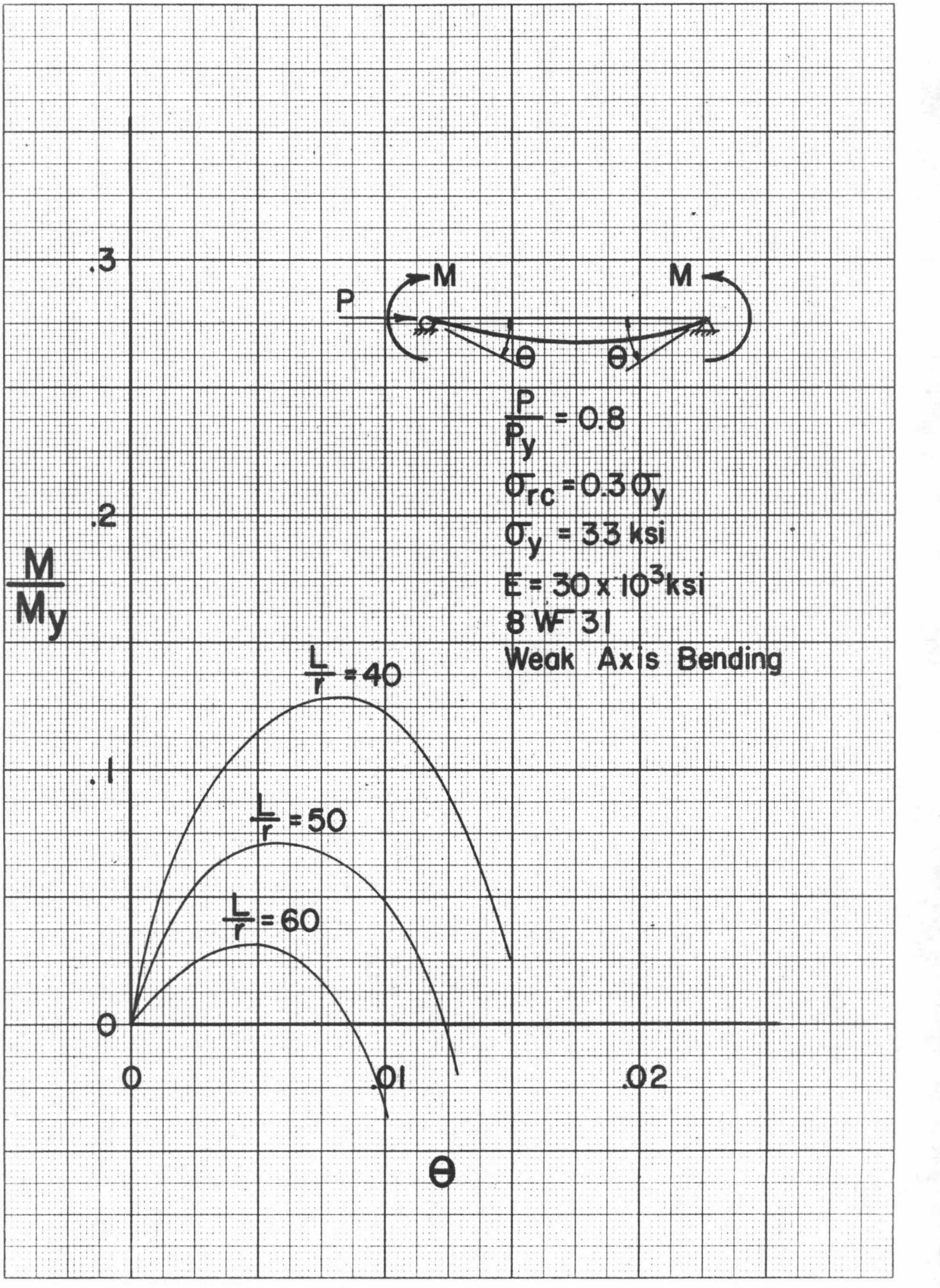
M
M
P
θ
θ
P/Py = 0.6
σrc = 0.3 σy
σy = 33 ksi
E = 30 x 10^3 ksi
8 WF 31
Weak Axis Bending
L/r = 40
L/r = 50
L/r = 60
L/r = 70
M/My
.6
.5
.4
.3
.2
.1
0
0
.01
.02
.03
.04
θ



P
M
M
θ
θ
P/Py = 0.7
σrc = 0.3 σy
σy = 33 ksi
E = 30 x 10³ ksi
8 WF 31
Weak Axis Bending
L/r = 40
L/r = 50
L/r = 60
L/r = 70
M/My
.3
.2
.1
0
0
.01
.02
θ



P
M
M
θ
θ
$\frac{P}{P_y} = 0.75$
$\sigma_{rc} = 0.3\,\sigma_y$
$\sigma_y = 33$ ksi
$E = 30 \times 10^3$ ksi
8 WF 31
Weak Axis Bending
$\frac{L}{r} = 40$
$\frac{L}{r} = 50$
$\frac{L}{r} = 60$
$\frac{L}{r} = 70$
$\frac{M}{M_y}$
.3
.2
.1
0
0
.01
.02
θ



M
P
M
θ
θ
P/Py = 0.8
σrc = 0.3 σy
σy = 33 ksi
E = 30 x 10^3 ksi
8 WF 31
Weak Axis Bending
L/r = 40
L/r = 50
L/r = 60
.3
.2
.1
0
M/My
0
.01
.02
θ

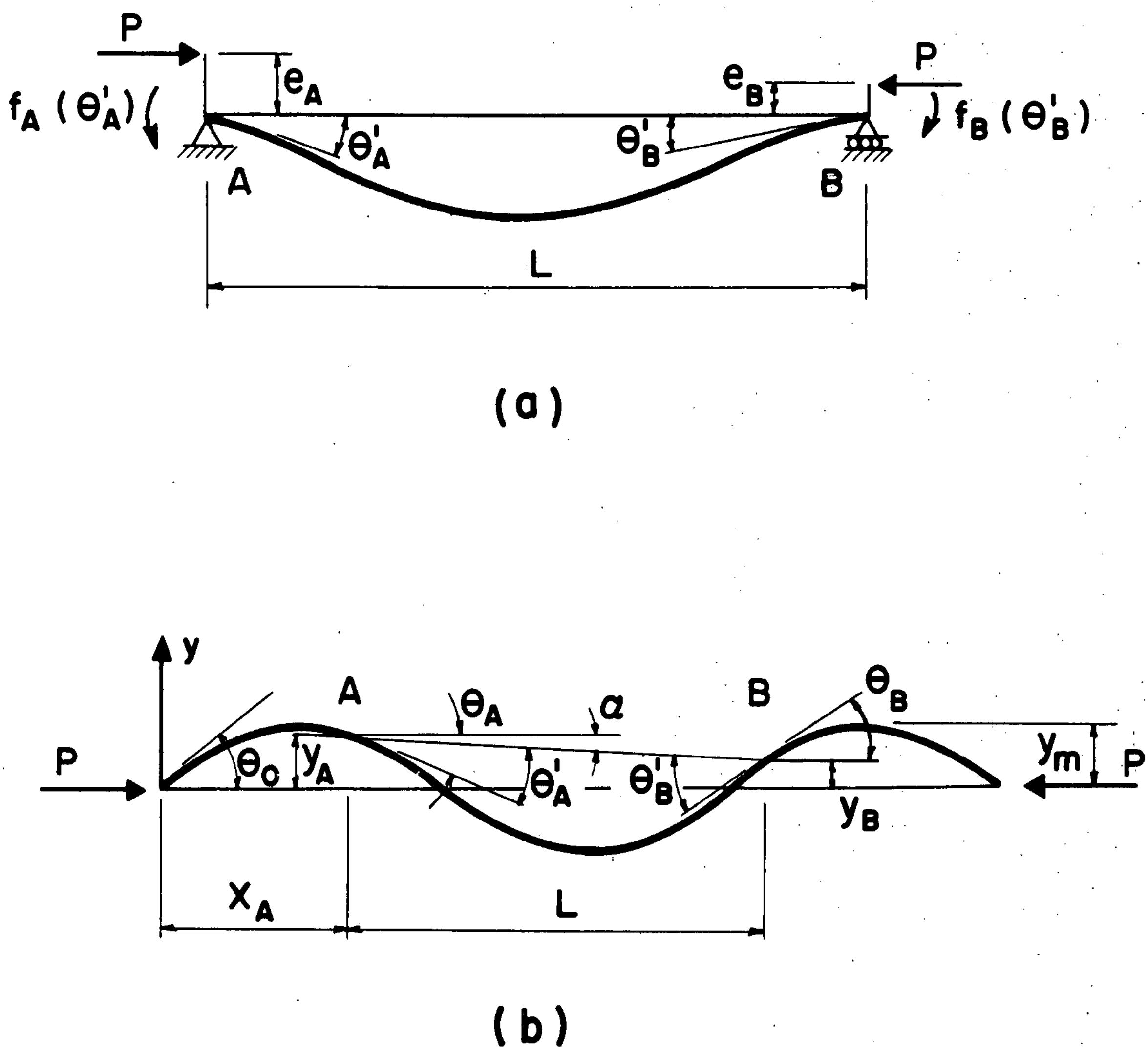


FIG. 1 GENERAL CASE OF BEAM-COLUMN LOADING

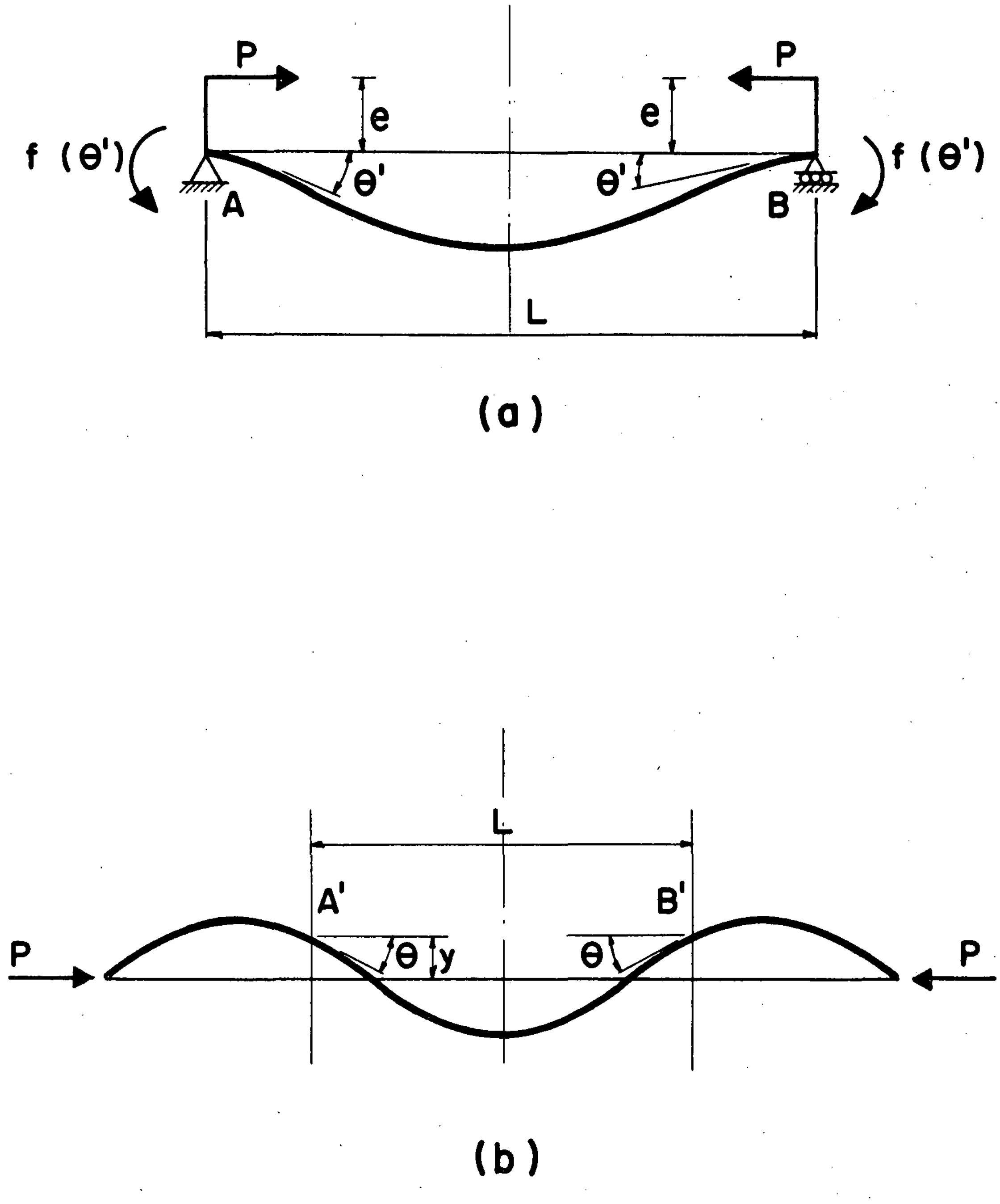


FIG. 2 SYMMETRIC BEAM-COLUMN

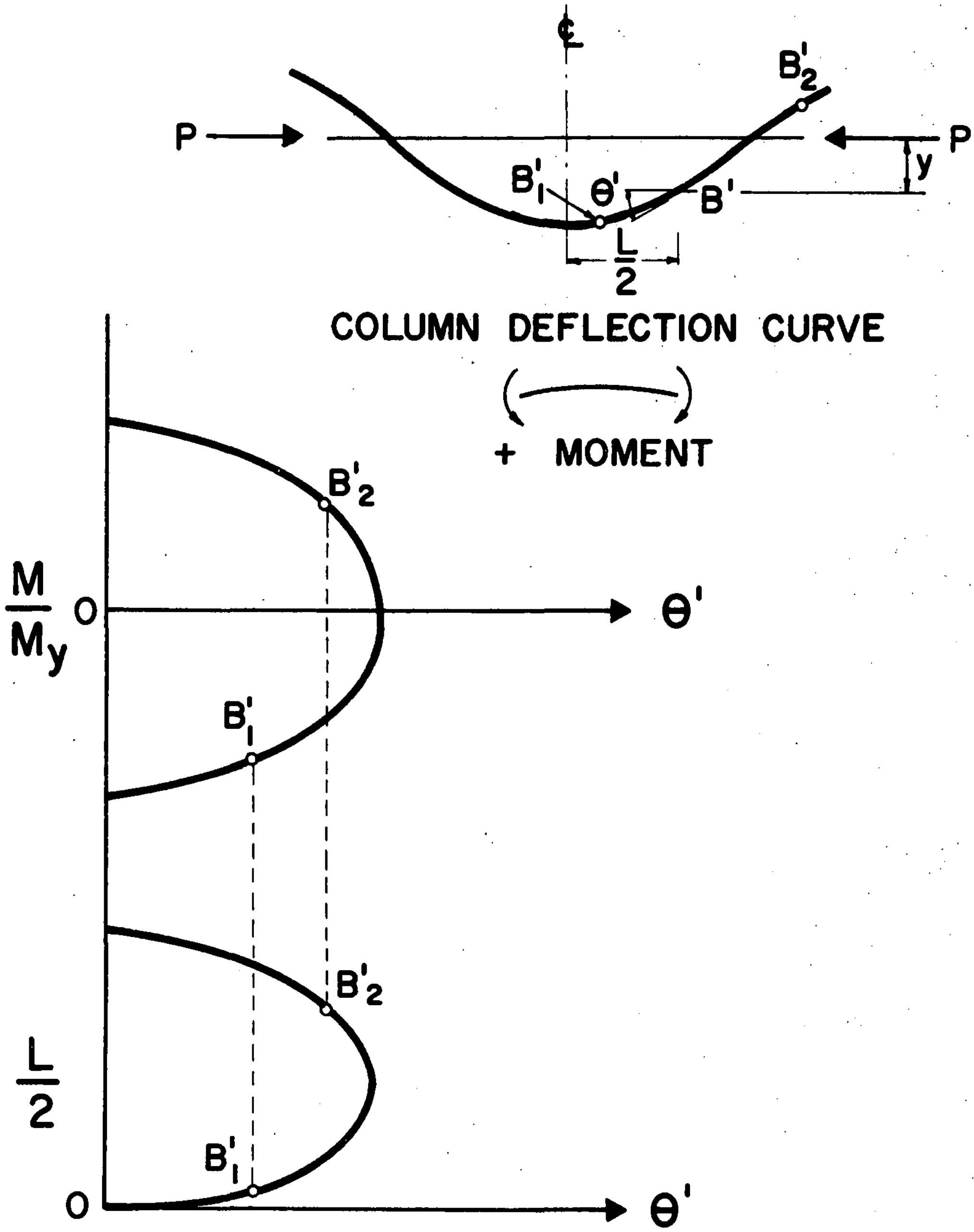
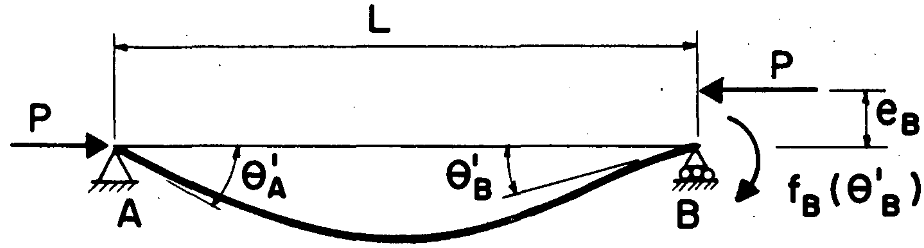
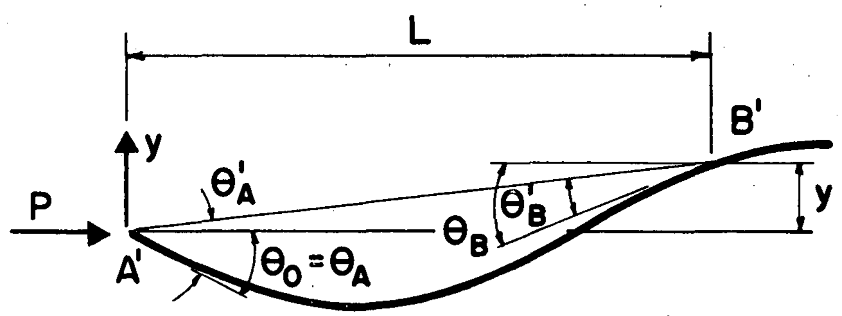


FIG. 3 MONOGRAPHIC REPRESENTATION OF THE COLUMN DEFLECTION CURVES



(a)



(b)

FIG. 4 PINNED-END BEAM-COLUMN

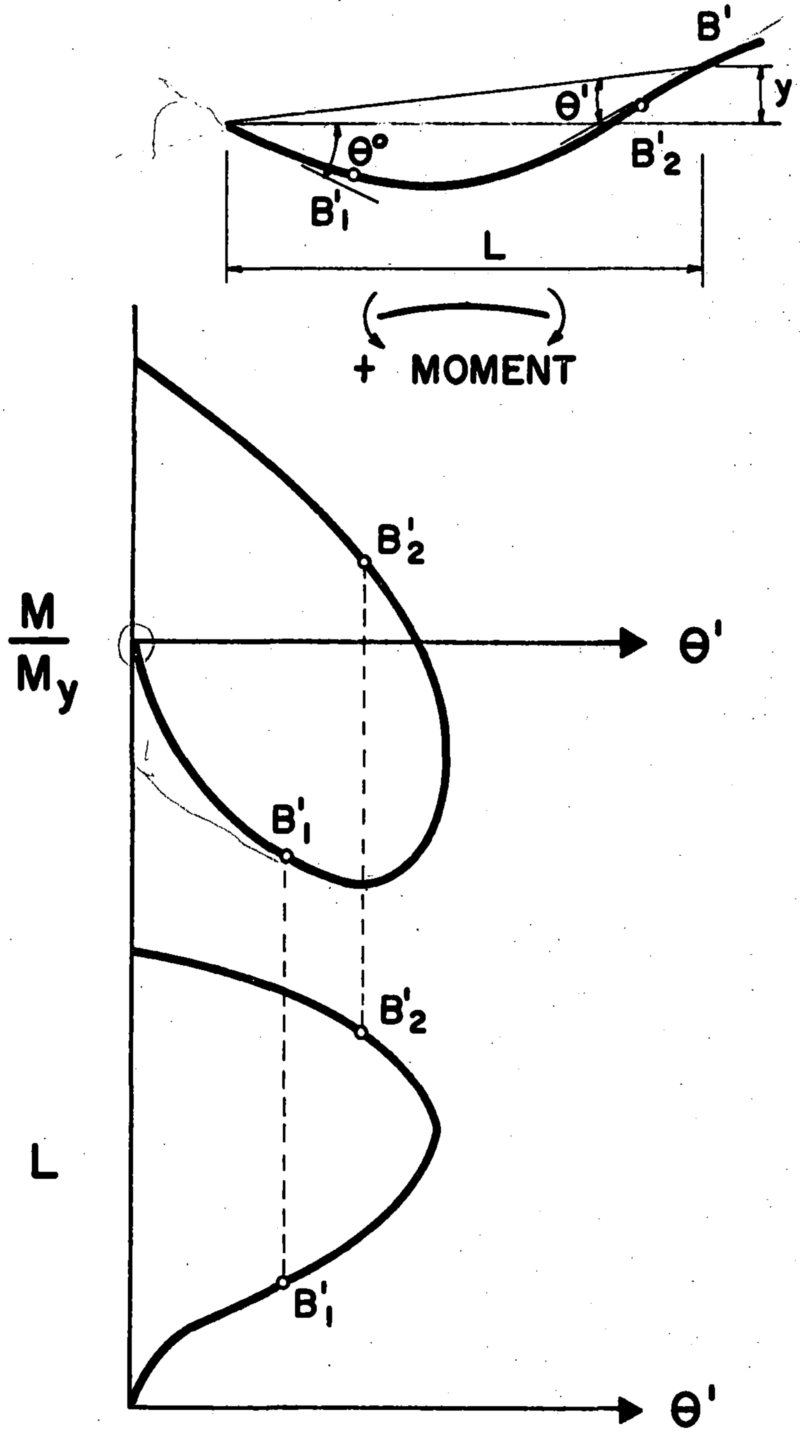


FIG. 5 MONOGRAPHIC REPRESENTATION OF THE COLUMN DEFLECTION CURVES

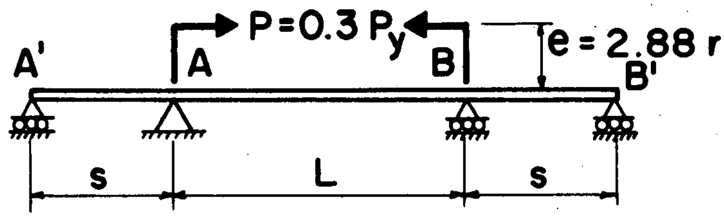


FIG. 6 PROBLEM III Ia

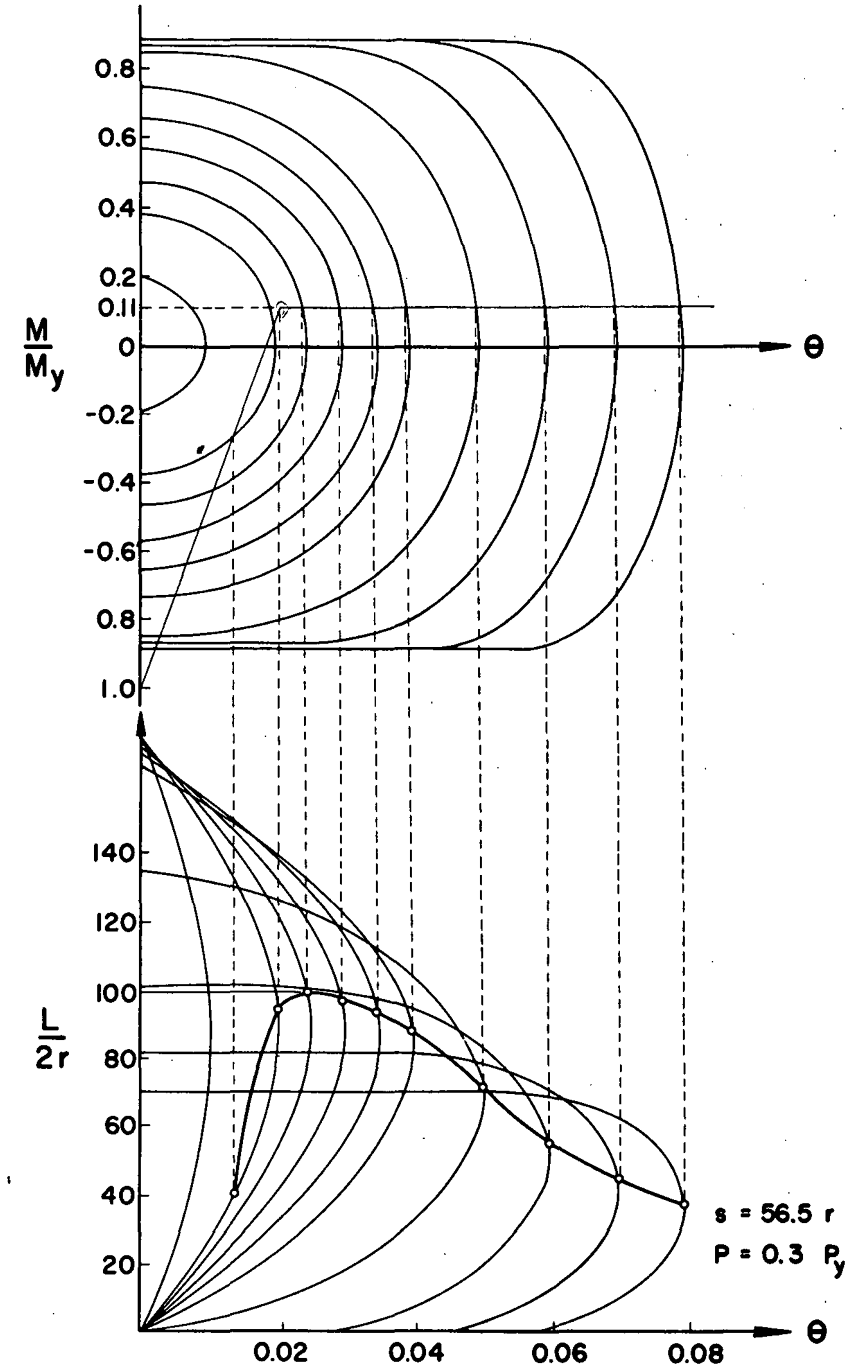


FIG. 7 SOLUTION OF III 1 (a)

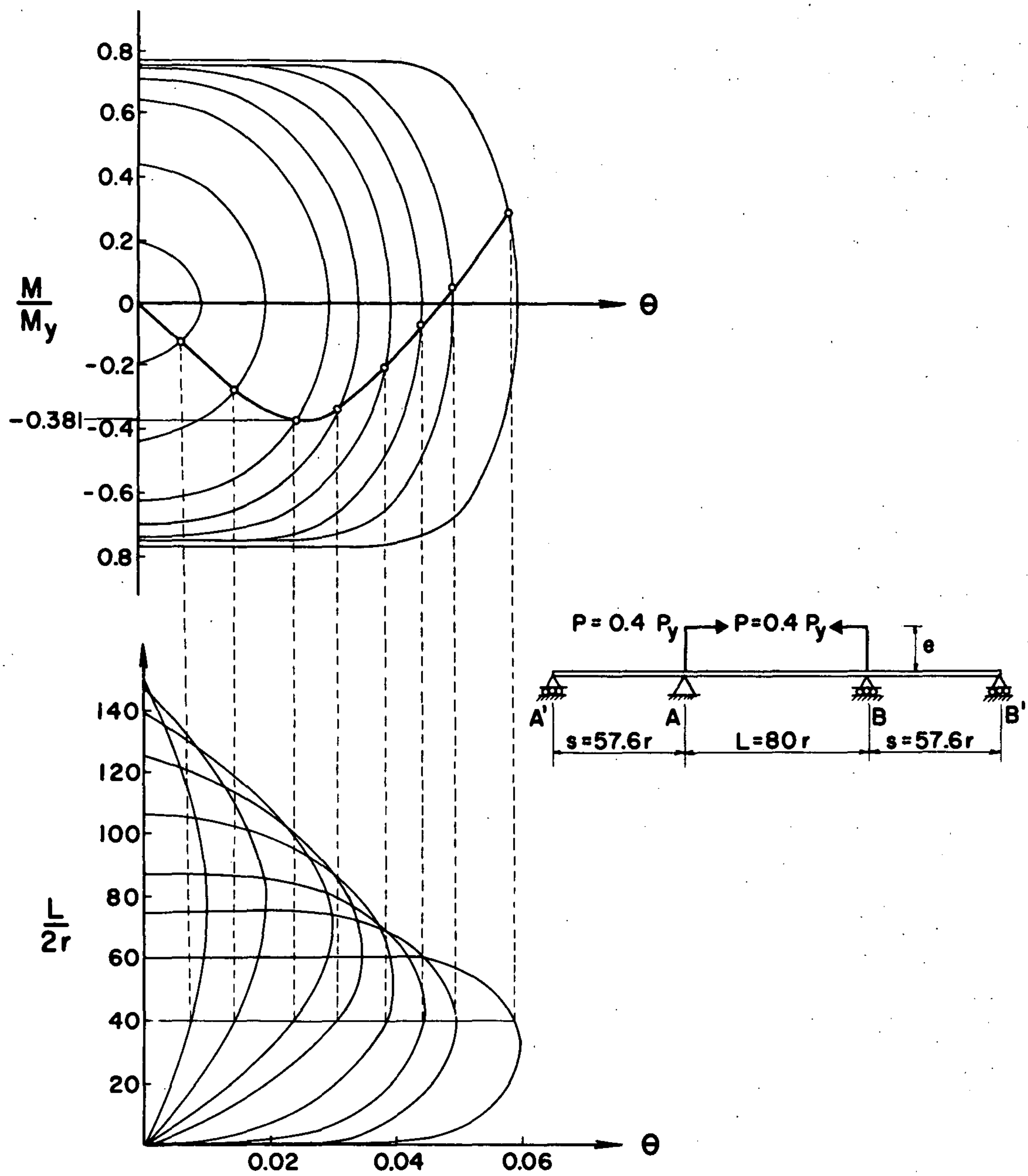


FIG. 8 SOLUTION OF III 1 (b)

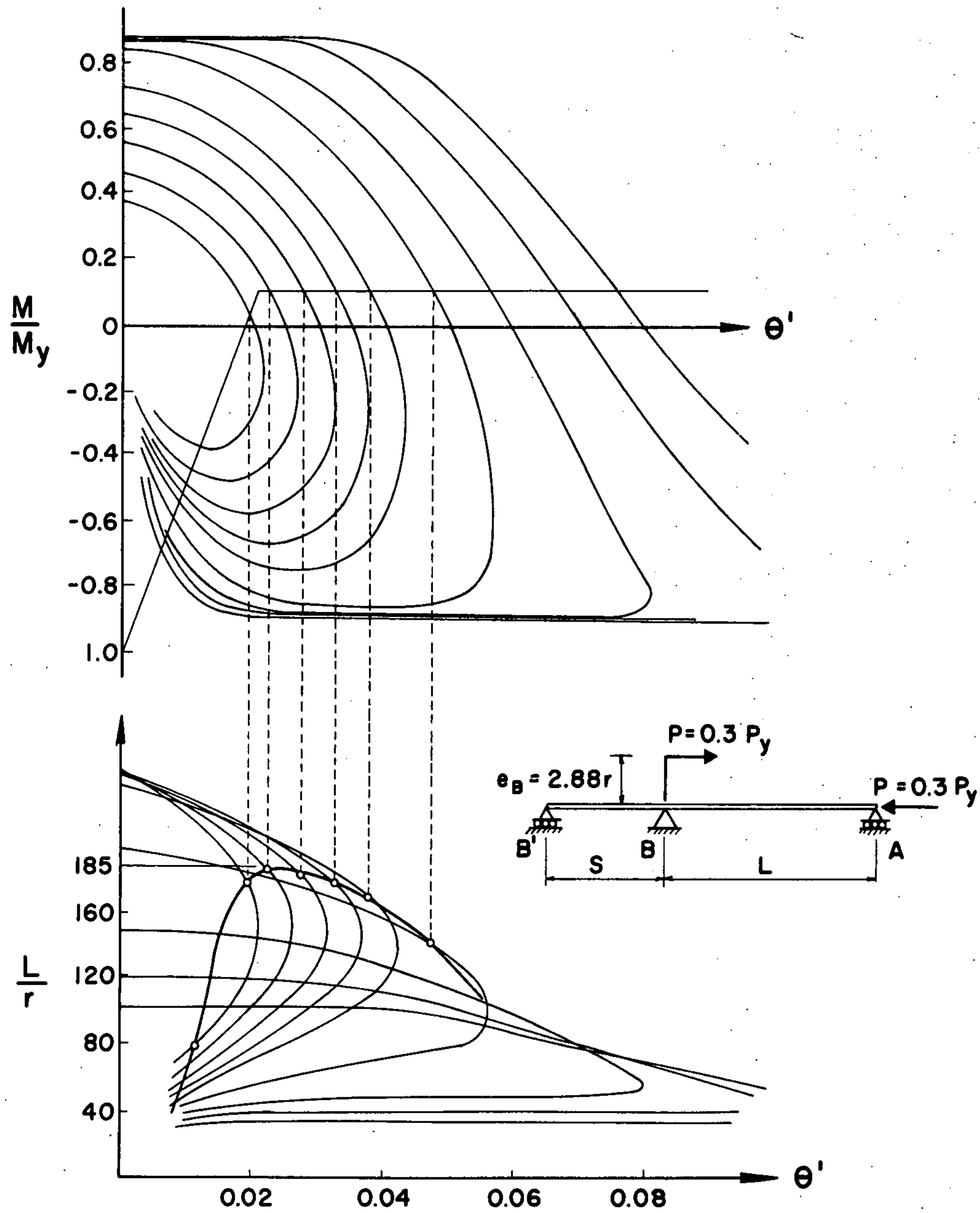


FIG. 9 SOLUTION OF III 2(a)

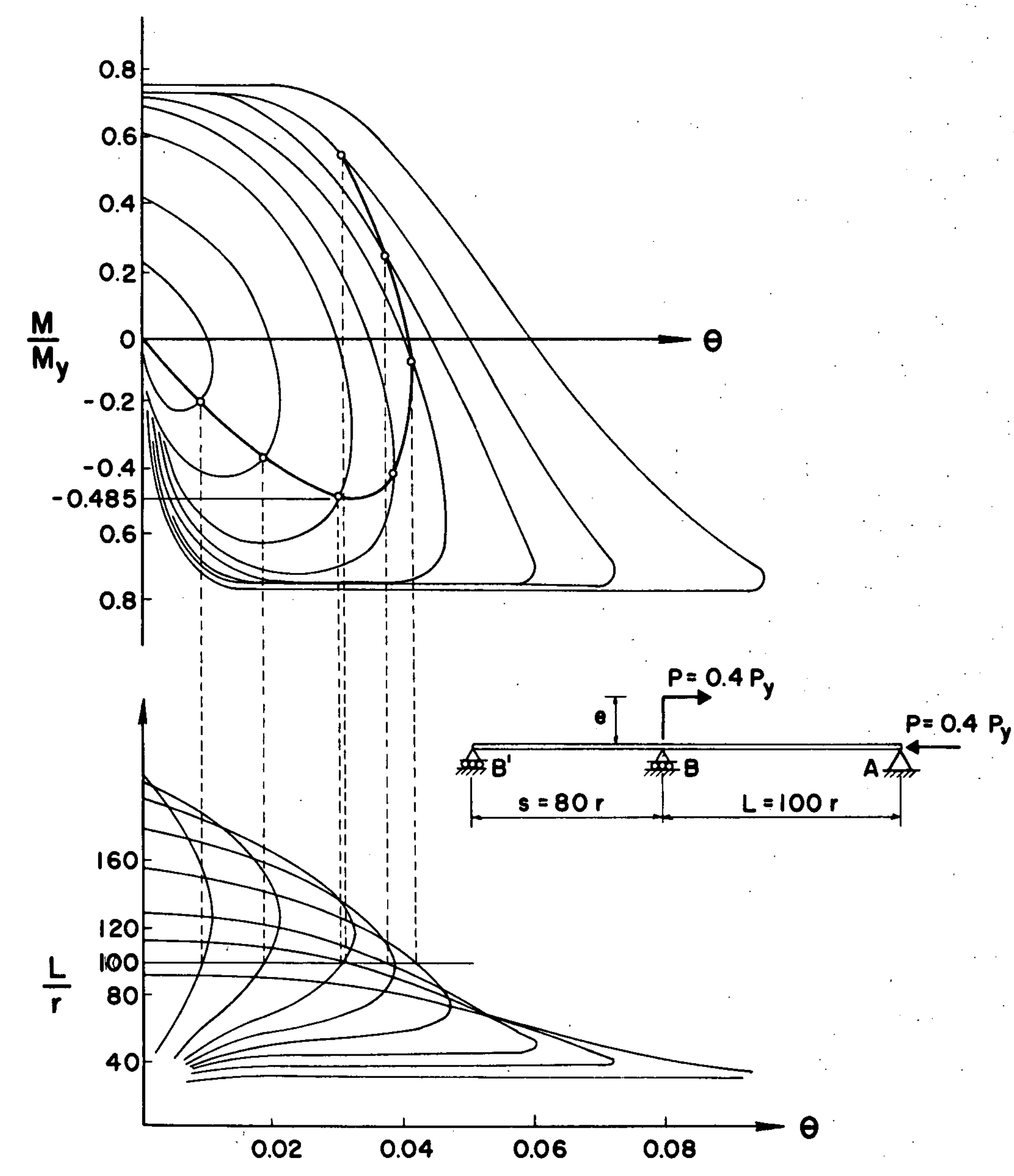


FIG. 10 SOLUTION OF III 2(b)

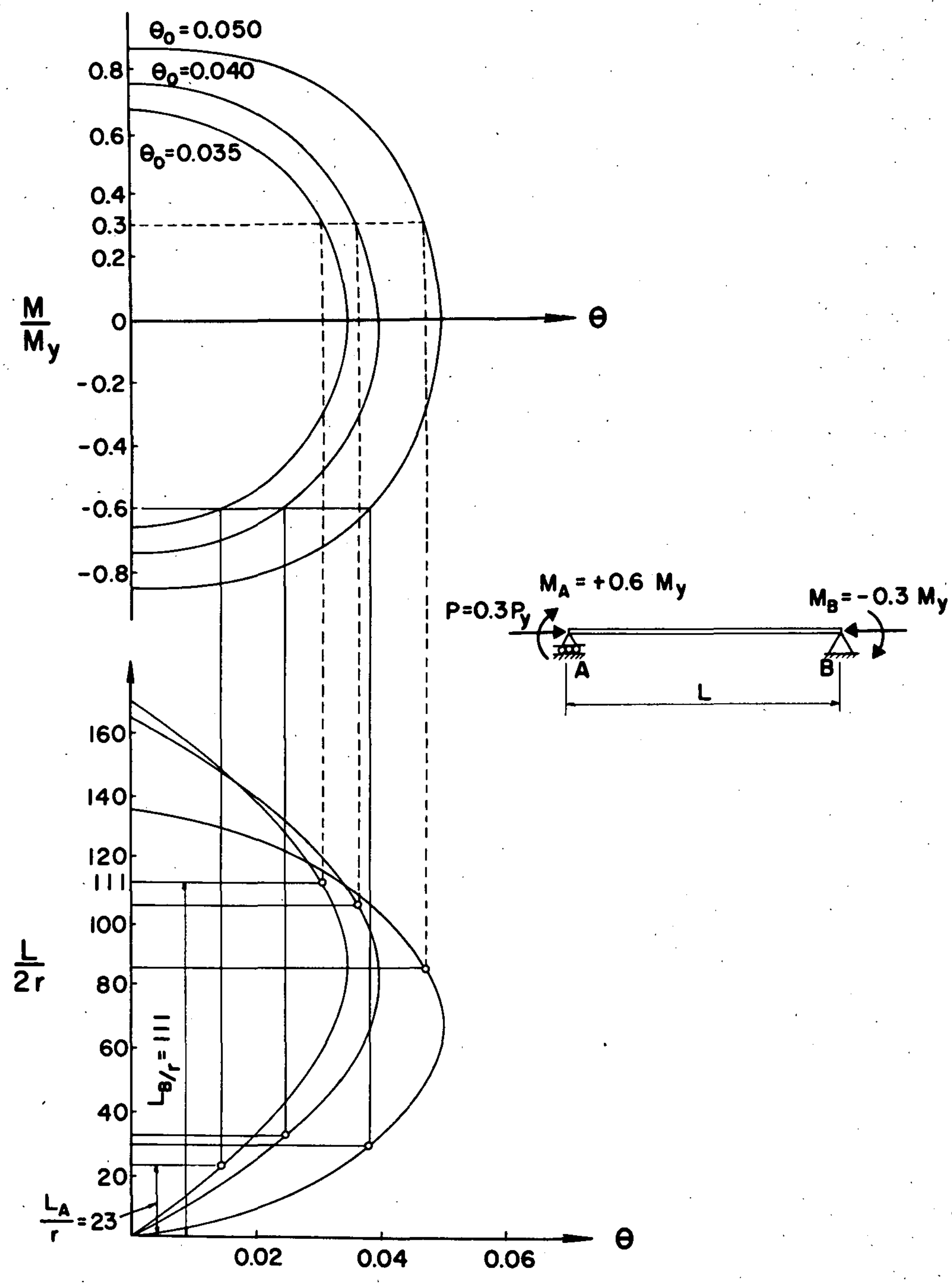


FIG. 11 SOLUTION OF III 3 (a)

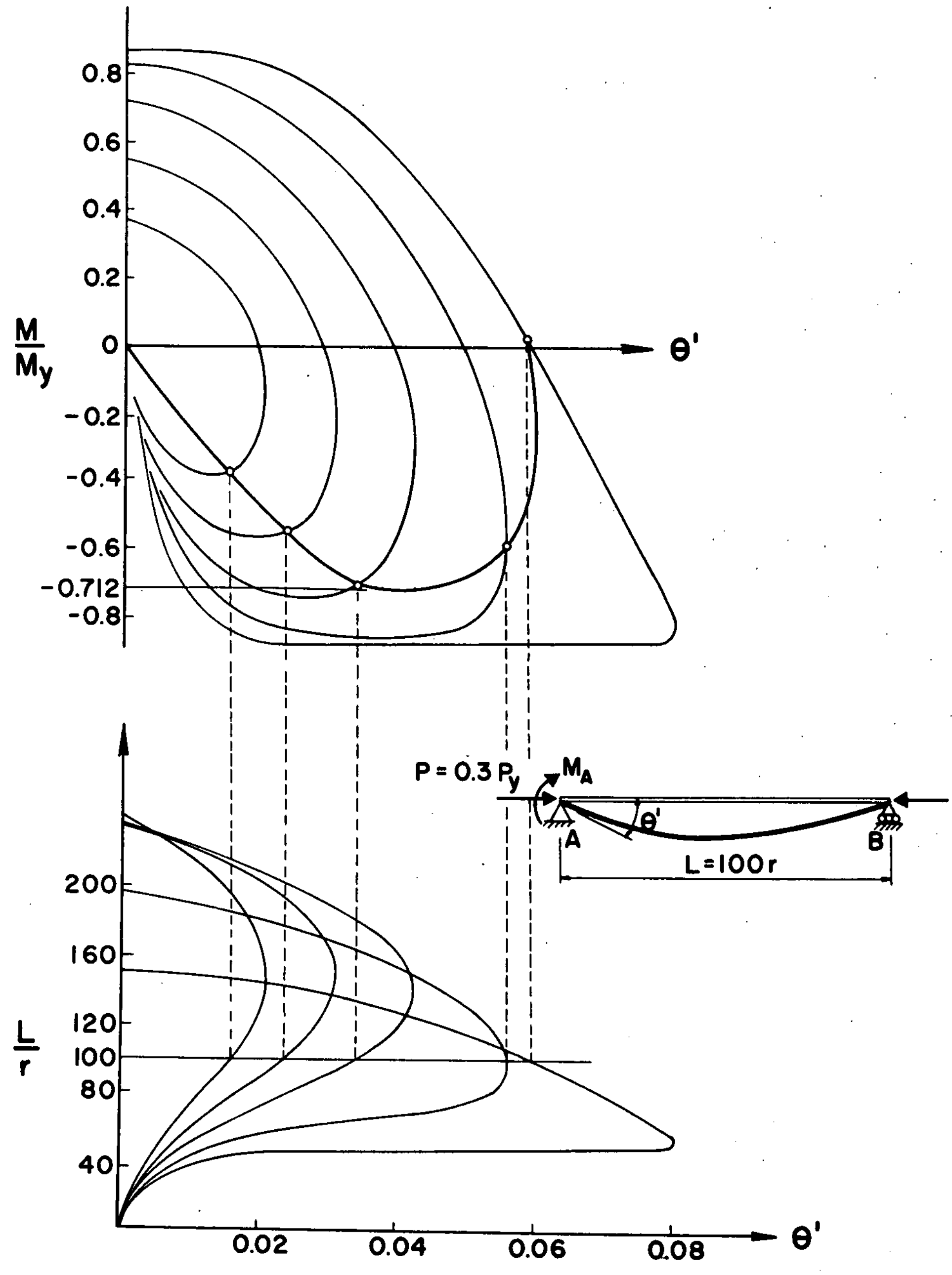


FIG. 12 SOLUTION OF III 3 (b)

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